Math Camp: Day 1

Scary Math Teacher!

School of Public Policy

George Mason University

January 13, 2014

6:00 to 8:00 pm

Lisardo Bolaños and **Lokesh Dani**

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Course Outline

Monday 1/13

- Estimating Costs
- Review of Math Camp
- Algebra Review
- Algebraic Functions
- Solving Linear Systems
- Coordinate geometry (JK)
- Exponents
- Compound Interest
- Non-linear functions
- Wrap-up, assignments
- Home Work! Yes!

Wednesday 1/15

- **Review assignment**
- Intro to Derivatives
- Intro to Optimization
- Intro to Probability (Kirk)
- Post-test
- Course assessment

Speed patrolling on the Mumbai - Pune Expressway

Estimating cost to patrol expressway for speeders

Total daily cost = cost of patrol cars + cost of labor + cost of gas

- **<u>Cost of Patrol Cars</u>**
- Given:
  - total distance one way = 200kms
  - Distance covered by one car in one patrol = 50 kms
  - Cost of leasing and maintaining a car per day = Rs. 7,500

- Daily cost of patrol cars = [(200*2)/(50)]*7500

Estimating cost to patrol expressway for speeders

- **<u>Cost of Labor</u>**
- Given:
  - 2 police per car
  - 3 shifts in a day, each of 8 hours
  - Police get paid Rs. 5,000 per day

- Daily cost of Labor = [(200*2)/50]*2*3*5000

Estimating cost to patrol expressway for speeders

- **<u>Cost of Gas</u>**
- Given:
  - Gas costs Rs.75/liter
  - Cars average 25kms/liter
  - Each shift on average completes 5 patrols
  - Each patrol covers 50kms
- Daily cost of gas = ({[(200*2)/50]*5*3*50}/25)*75

Estimating cost to patrol expressway for speeders

- **Daily cost = cost of patrol cars + cost of labor + cost of gas**
- Checking the units:
- $$\frac{200\ \cancel{kms} * 2}{50\ \cancel{kms}} * Rs.\ 7500 + \frac{200\ \cancel{kms} * 2}{50\ \cancel{kms}} * 2 * 3 * Rs.5000$$

$$+ \frac{\frac{200\ \cancel{kms} * 2}{50\ \cancel{kms}} * 5 * 3 * 50\ \cancel{kms} * Rs.75/\cancel{liter}}{25\ \cancel{kms}/\cancel{liter}}$$

- Final units will be in Rs.

Estimating cost to patrol expressway for speeders

- Daily cost =

  [(200*2)/50] * 7500 +

  [(200*2)/50] * 2*3*5000 +

  ({[(200*2)/50]* 5*3*50}/25)*75 = ?

  8 * 7,500 +

  8 * 2*3*5,000 +

  ((8 * 5*3*50)/25)*75 = ?

P	Parentheses
E	Exponents
M	Multiplication
D	Division
A	Addition
S	Subtraction

8 * 7,500 + 8 * 30,000 + 8 * 2,250 = 8 * (7,500 + 30,000 + 2,250)

= 8 * 39,750

= <u>Rs. 318,000</u>

Is it worth patrolling?

- Given:
  - Based on toll collection, on average 150,000 cars use expressway daily. Currently 30 percent of cars go over speed limit. However, estimates suggest with patrolling only 6 percent of cars will risk speeding.
- How many cars will risk speeding?
- If patrolling is only effective enough to catch 7 of 10 speeders, how many speeders would be ticketed daily?
- If each speeding fine is Rs.300 what would be the average daily revenue from patrolling speeders?
- If police get paid Rs. 5,000/day, approximately how many speeding tickets would a corrupt cop have to pocket to double their daily wage?

Speed Patrolling on the Mumbai - Pune Expressway

Current Solution

- Time taken = Distance Covered / Average Speed
- Speed limit : 80 km/hr
- Total Distance: 200 km
- Time taken at average speed limit = (200km) / (80 km/hr)

  = "x" hours
- What is the value of "x"?

If car takes less than "x" hours from start toll booth to the end toll booth, driver is fined for speeding.

Why Math Camp?

Purpose

- Prep for Economics and Statistics
- Get thinking in "math mode" again
- Develop familiarity with step-wise approach to solving problems
- Improve confidence and get excited about math

Structure

- Self-assessment
- Learning path
- Khan Academy & Online Modules
- In-class sessions
- Post math camp online resources
- E-booklet of solved problems

Algebra Review

Concepts and Notation

- **Variable:** a symbol for a number that can change (often use x and y)
- **Constant:** a number that does not change
- **Coefficient:** a number used to multiply a variable

Coefficient → 4; Variable → x; Constants → 7, 5

$$4x - 7 = 5$$

Algebra Review

- **Solve for x**
- Combine x terms on one side, constants on the other side by adding and subtracting on both sides of the equation
- Divide by the coefficient (on both sides) to isolate x

- $2x - 4 = 8 - x$
- $+x \qquad\qquad +x$
- $3x - 4 = 8$
- $+4 \qquad +4$
- $3x = 12$
- $\overline{3} \qquad\qquad \overline{3}$
- $x = 4$

Algebra Review

- **<u>Exercises</u>**
- **Solve for x**
- **$7x - 12 = 3x + 8$** **$\frac{1}{2} x - 1 = \frac{1}{3} x + 5$**

Algebraic Functions

Algebraic Functions

- Understanding functions is critical to defining relationships between inputs and outputs.

- **Function:** relates one quantity or input with one and only one quantity or output
- e.g. $\mathbf{y = f(x) = 3x^2 + 2x + 8}$

  - **<u>Input</u>**: independent variable, exogenous variable, *x* or *t*
  - **<u>Output</u>**: dependent variable, endogenous variable, *y*

  So, *y* equals some function of *x* (or *t*) → *y = f(x)*

  f is often used as the notation for a function but it does not have to be used → *g(x), v(t)*

We map relationships in terms of functions

Outputs : LHS **Inputs : RHS**

c(patrolling) = c(patrol cars) + c(labor) + c(gas)

(daily cost of patrolling is a function of the cost of the cars, the cost of labor, and the cost of gas)

Outputs : LHS **Inputs : RHS**

Profit = Total Revenue – Total Costs

Often denoted as $\pi = I - C$

(profit is a function of revenue and costs)

Functions

- **What is a function?**

A function is a one to one mapping of an element from one set to <u>one and only one</u> element of another set.

A function cannot have 2 y-values for one x-value

- **What is NOT a function?**

The equation of a circle is an example of NOT a function.

$x^2 + y^2 = 1$ **OR** $y = \sqrt{(1 - x^2)}$

because one value of x maps to 2 values of y.

Such equations are called a "**correspondence**".

Linear Functions

Coefficient — Constant

$$f(x) = mx + b$$

or

$$y = mx + b$$

SLOPE — Y-intercept

Linear Functions

Slope Y-intercept

$$f(x) = y = mx + b$$

Variables

Example

$f(x) = \frac{2}{3}x - 4$

Evaluate the following when $x = 6$?

Or what is the value of the function $f(6)$?

$$f(6) = \tfrac{2}{3}(6) - 4$$
$$= 12/3 - 4$$
$$= 4 - 4 = 0$$

Exercise

Given $v(t) = 2t + 4$

What is the value of the function $v(10)$?

Linear Functions

- How would you write a linear function f(x) for the following scenario?
- ***In the School of Public Policy, the tuition for each course credit is $678 (in-state.)***

Let x be the number of course credits. Then;

$f(x) = 678*x$ *OR* $f(x) = 678*x + 0$

slope = 678, y-intercept = 0

- ***What is the cost for a 3 credit course?***

f(3) = 678(3) = 2,034

Solving Linear Systems

Solving Linear Systems

Solving a system of two equations with two variables

I. $y = 3x - 1$

II. $y = 2x$

- Substitute $y = 2x$ into equation 1

  $2x = 3x - 1$

  $1 = x$

I. $10x - 5y = 20$

II. $x + y = 2$

- Re-arrange equation 2

  $y = 2 - x$

- Substitute into equation 1

  $10x - 5(2 - x) = 20$

  $10x - 10 + 5x = 20$

  $15x - 10 = 20$

  $15x = 30$

  $x = 2, y = 0$ (as $y = 2 - x$)

Solving Linear Systems: 2 equations

Exercises

Solve for x and y

- **$3x - y = 2$ ………………….(1)**
- **$2x + 2y = 12$……………….(2)**

Always check your answers by substituting values of x and y back into the equations.

Solution

From (2) we get

$2x + 2y = 12$

$2(x + y) = 12$

$x + y = 6$

$y = 6 - x$ …………………… (3)

Substituting (3) in (1)

$3x - (6 - x) = 2$

$3x - 6 + x = 2$

$4x - 6 = 2$

$4x = 8$

$\underline{x = 2}$ ……………………..(4)

Substituting (4) in (3)

$\underline{y = 4}$

Solving Linear Systems: 3 equations

Solve for x, y and z

- $3x - y + z = 2$……………………(1)
- $2x + 2y - z = 12$………………..(2)
- $x + y - z = 6$ ……………………..(3)

Always check your answers by substituting values of variables back into the given equations.

Solution

Add (1) and (3)

$4x = 8$

$\underline{x = 2}$ …………………………..(4)

Substitute (4) in (2)

$2y - z = 8$ ………………………… (5)

Substitute (4) in (3)

$y - z = 4$…………………………..(6)

Subtract (6) from (5)

$\underline{y = 4}$ …………………………..(7)

Substitute (4) and (7) into (3)

$\underline{z = 0}$

Coordinate Geometry

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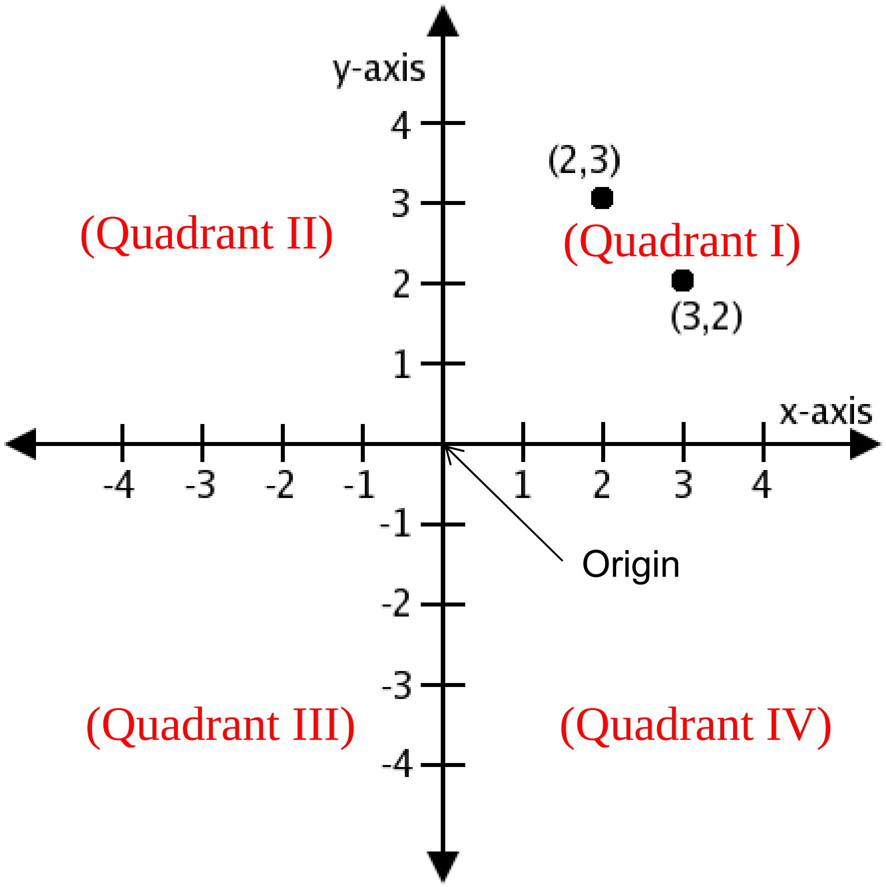
January 13, 2014

Module V
https://sites.google.com/site/sppmathcamp/module-v

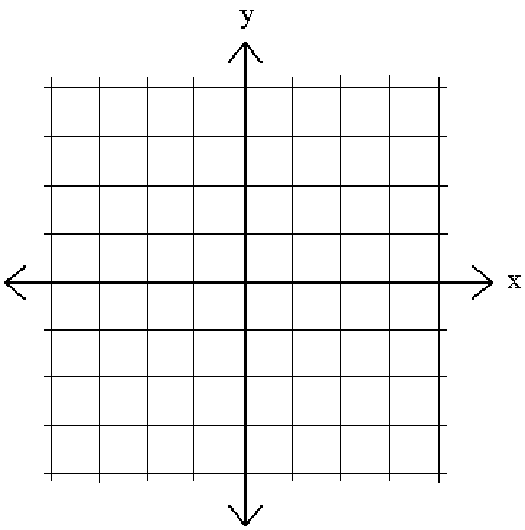
Coordinate Plane



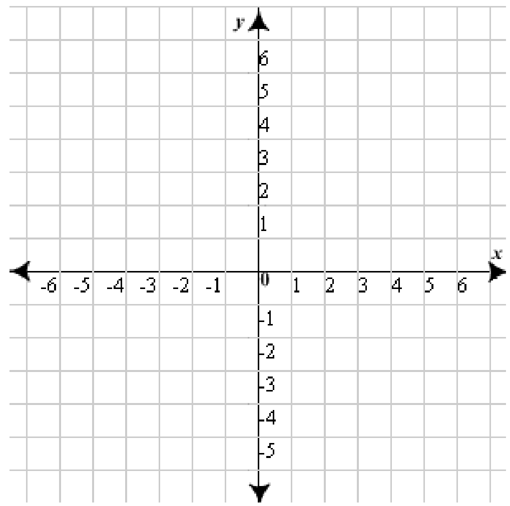
The “coordinate” of any point can be written as an ordered pair.

(x value, y value)

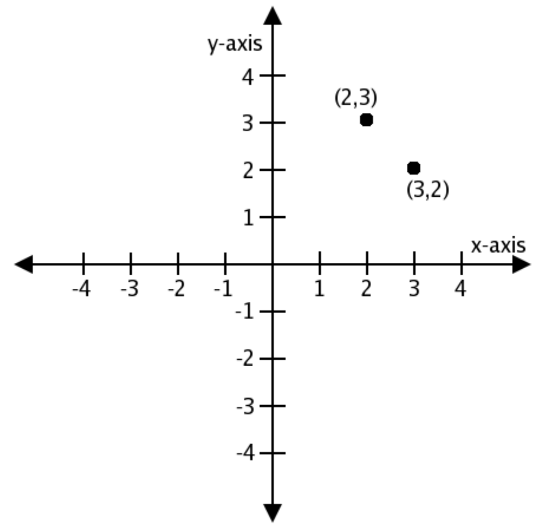
(a)

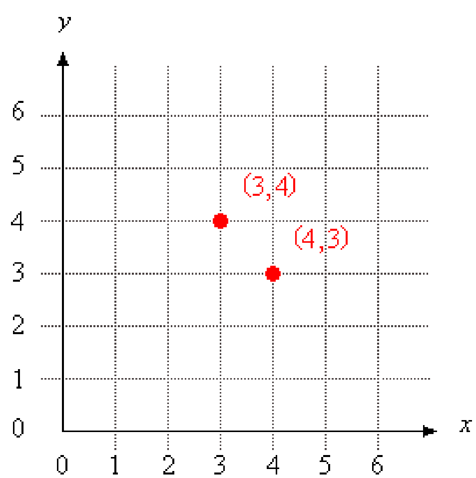

y
x

(b)

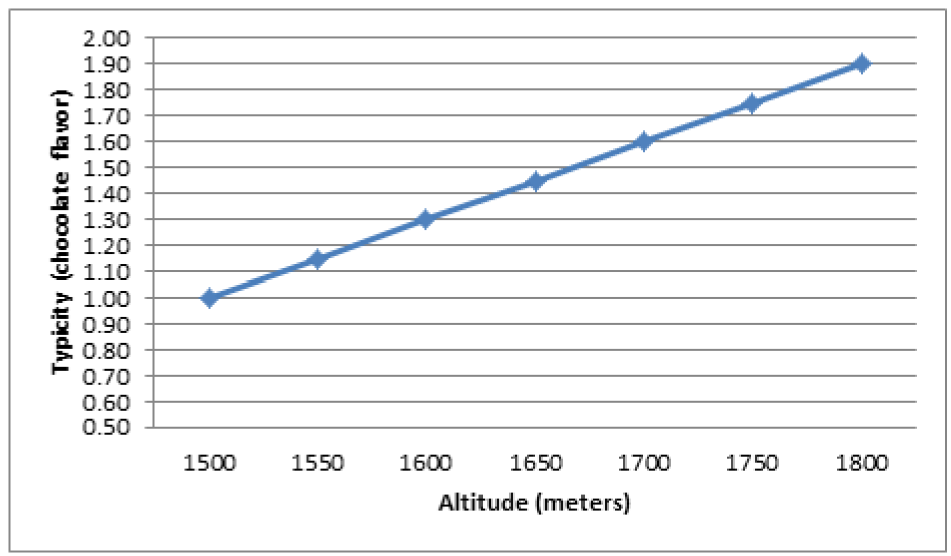

y
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
x
-6 -5 -4 -3 -2 -1
1 2 3 4 5 6

(c)


y-axis
4
3
2
1
-1
-2
-3
-4
(2,3)
(3,2)
x-axis
-4 -3 -2 -1
1 2 3 4


y
6
5
4
3
2
1
0
(3,4)
(4,3)
x
0 1 2 3 4 5 6

(d)


Typicity (chocolate flavor)
2.00
1.90
1.80
1.70
1.60
1.50
1.40
1.30
1.20
1.10
1.00
0.90
0.80
0.70
0.60
0.50
1500 1550 1600 1650 1700 1750 1800
Altitude (meters)

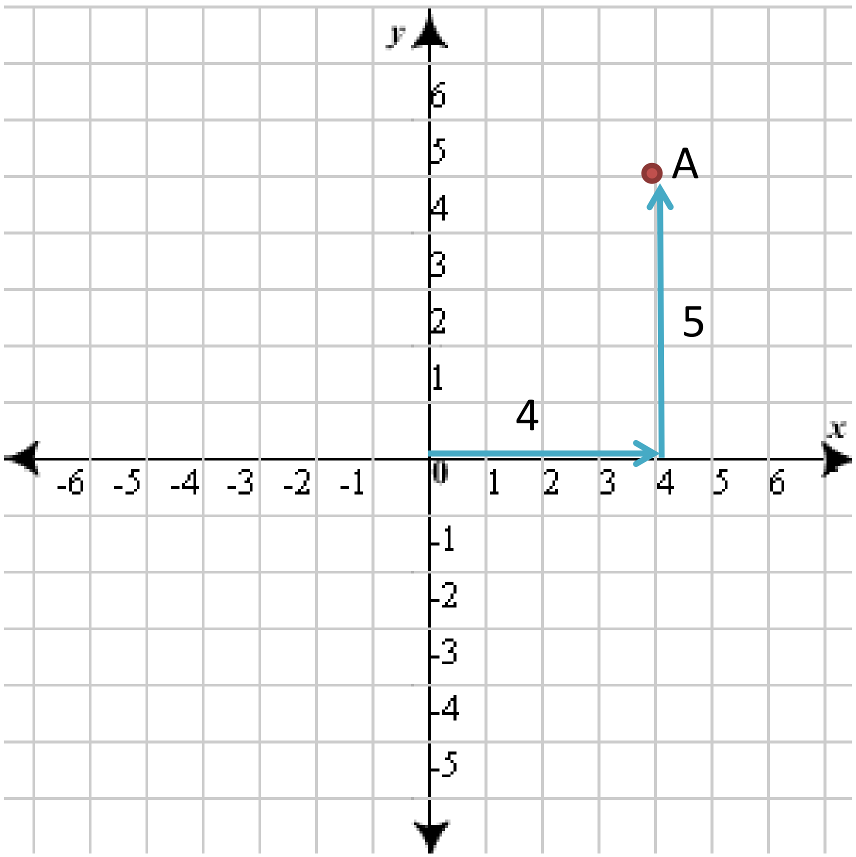
(e)

Ordered Pairs

Any ordered pair constitute a ***relation*** between *y* and *x*.

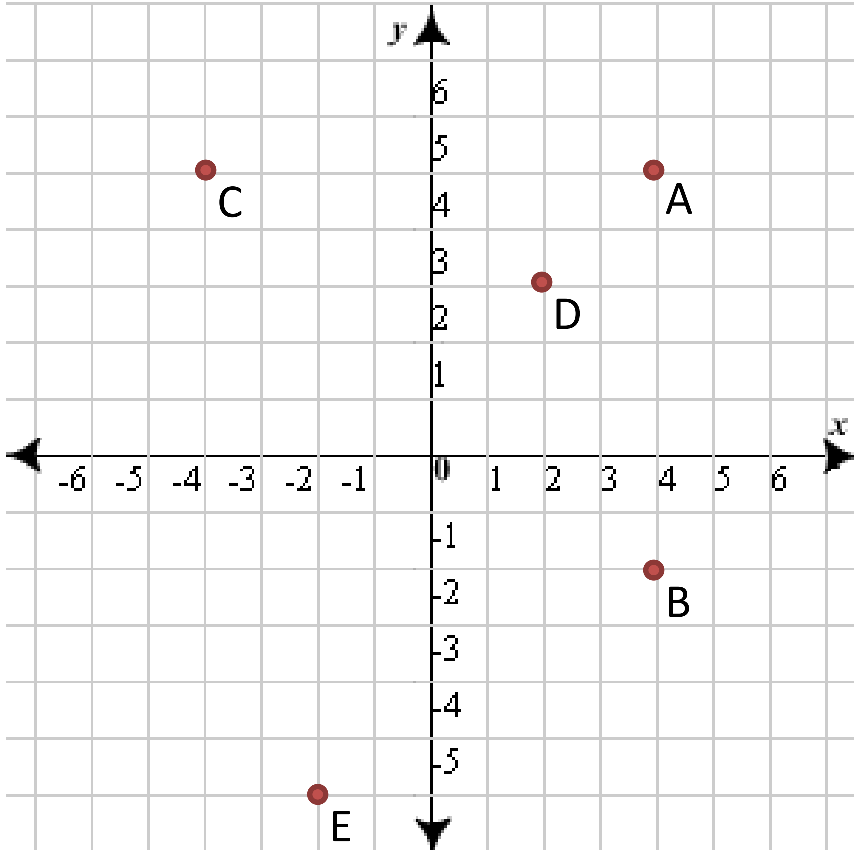
- (a, b)
- (x value, y value)
- (value for the horizontal-axis variable, value for the vertical –axis variable)
- (input, output)
- (altitude, chocolate flavor)

Plotting Point



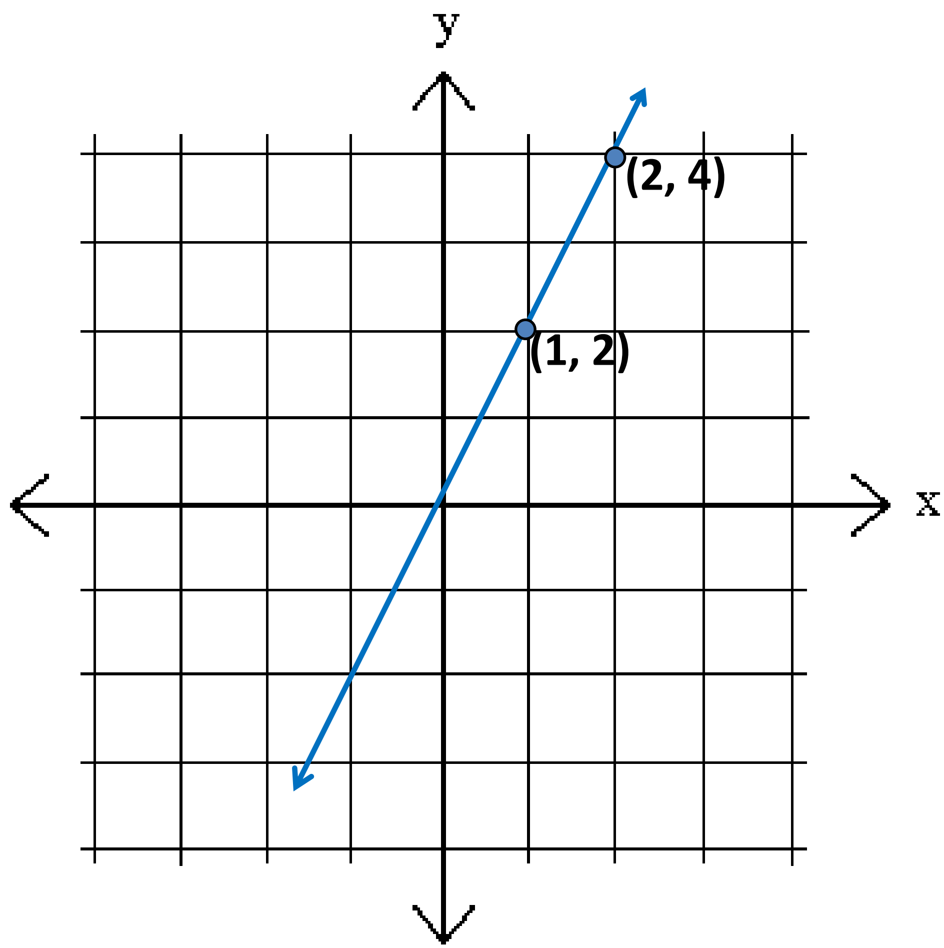
- A = (4, 5)

Coordinate Plane & Ordered Pairs: Exercise



- A = (4, 5)
- B = (4, -2)
- C = (-4, 5)
- D = (2, 3)
- E = (-2, -6)

Function & Ordered Pairs

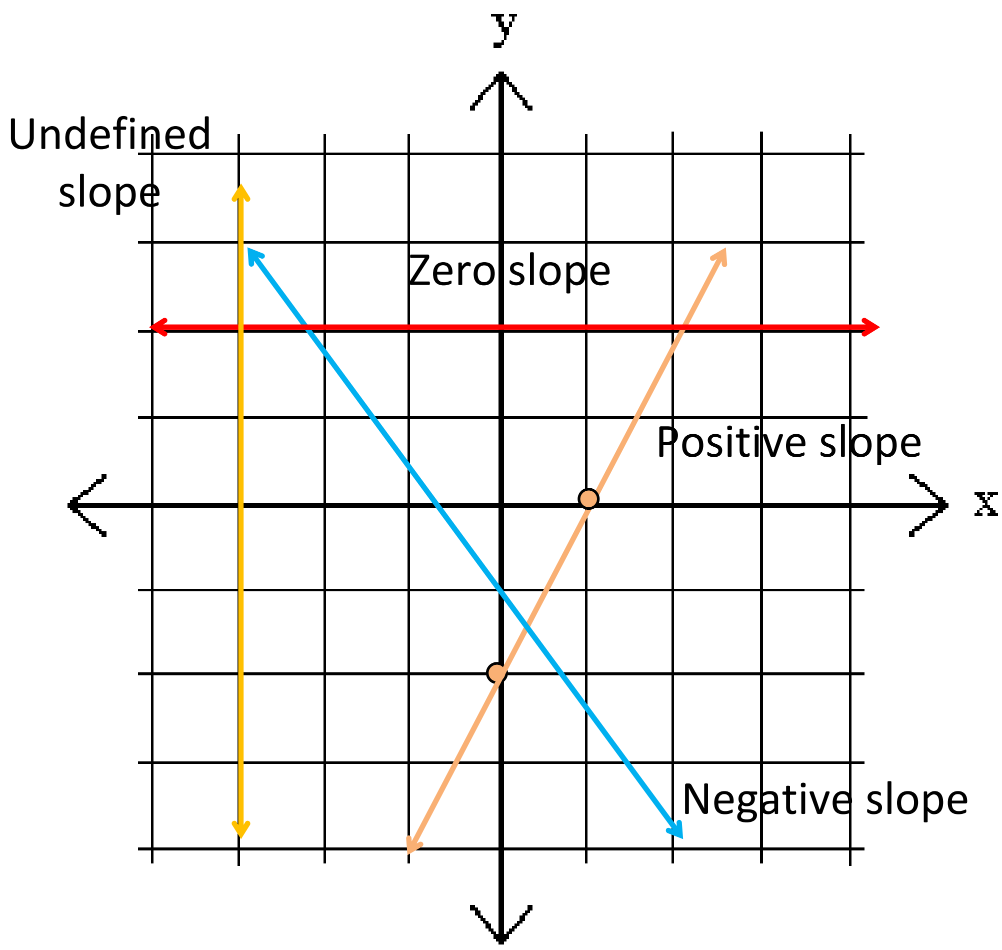


- A function is a set of ordered pairs showing relation between *y* and *x*.
- slope-intercept form:

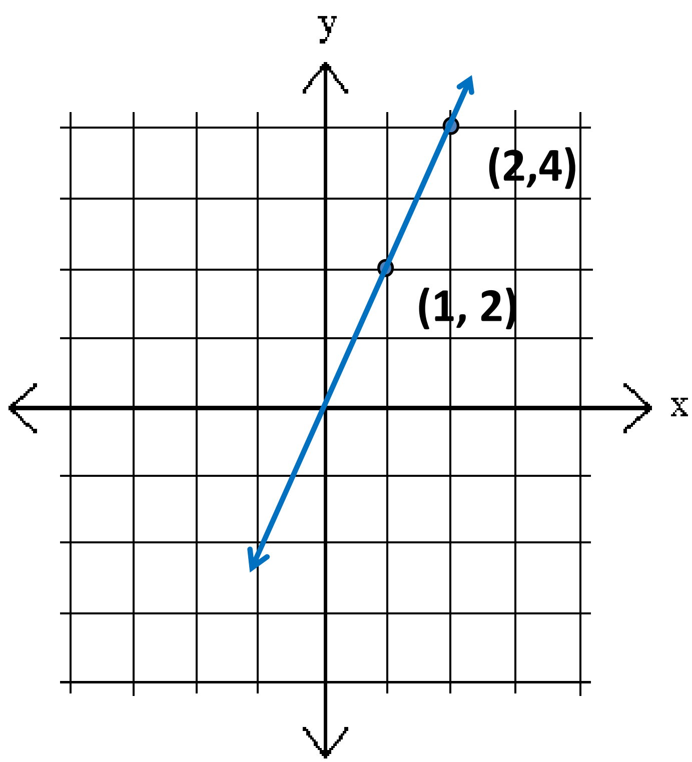
$$y = mx + b$$

↑ slope ↑ y-intercept

Coordinate Geometry



Graph & Function



(1) Write down the slop-intercept form of the function.

$$y = mx + b$$

(2) Plot any two points:

$$(1, 2) \text{ and } (2, 4)$$

(3) Calculating slope *m* "rise over run":

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{4 - 2}{2 - 1} = 2$$

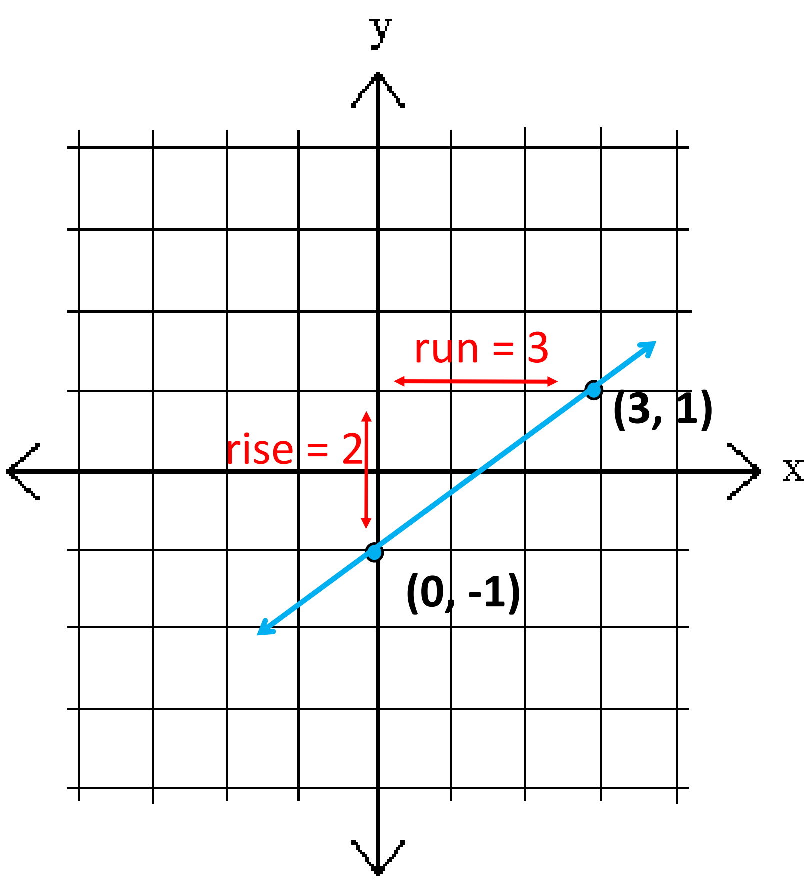
(4) Find y-intercept (value of y when x=0)

$$\text{y-intercept} = 0$$

(5) Complete the function.

$$y = 2x + 0 = 2x$$

Graph & Function: Exercise (1)



(1) Write down the slop-intercept form of the function.

$$y = mx + b$$

(2) Plot any two points:

(0, -1) and (3, 1)

(3) Calculating slope m "rise over run":

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-1)}{3 - 0} = \frac{2}{3}$$

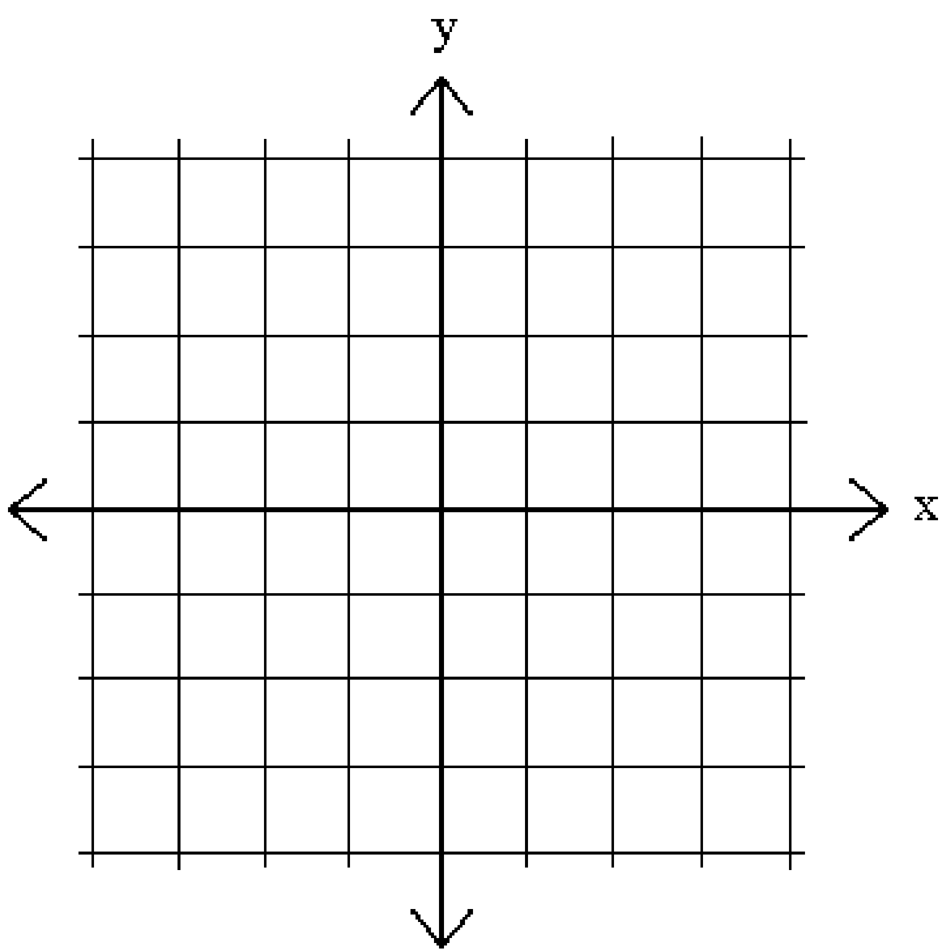
(4) Find y-intercept (value of y when x=0)

When $x = 0, y = -1$

(5) Complete the function.

$$y = \frac{2}{3}x - 1$$

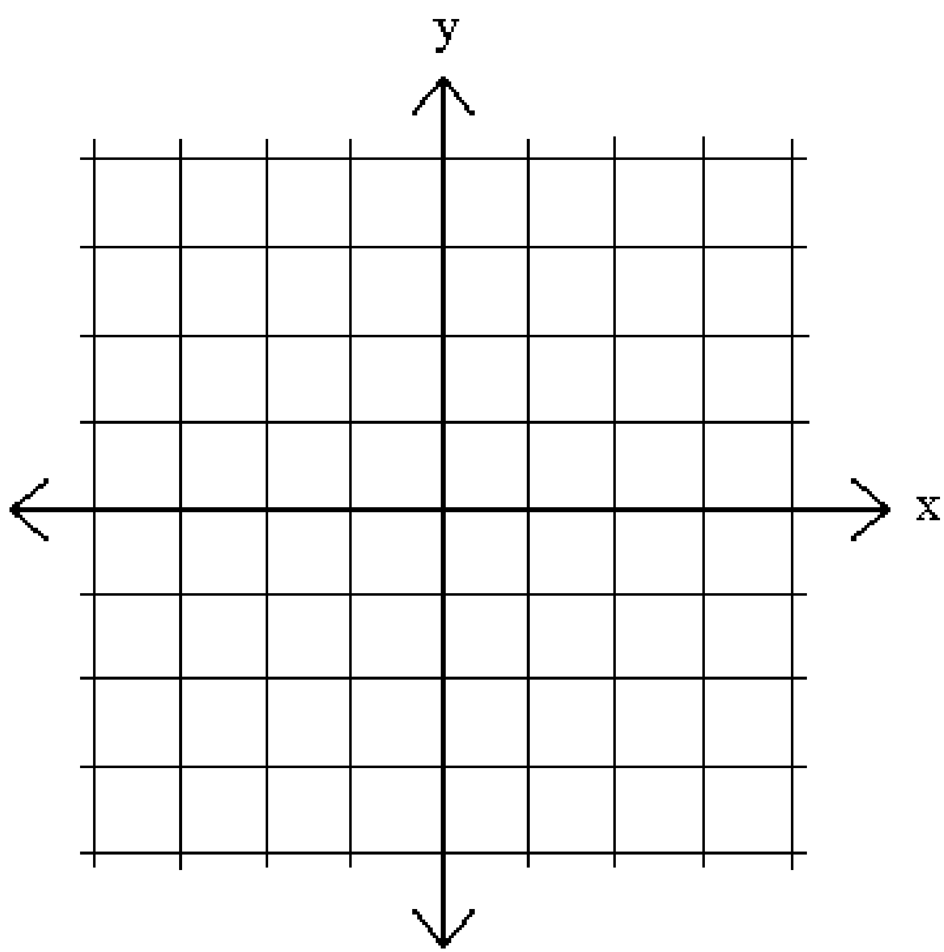
Graph & Function: Exercise (2)



Graph the function:

$$y = 3 - 2x$$

Graph & Function: Exercise (3)



Graph the function:

$$4x - 2y = 4$$

Exponents

Exponents: Rules

- $z^x = z*z*z*z$ … *(x times)*

  e.g. $4^3 = 4 * 4 * 4 = 64$

- $z^x * z^y = z^{(x+y)}$

  e.g. $4^2 * 4^3 = 4^{(2+3)}$ *Must be the same base number*

- $z^x / z^y = z^{(x-y)}$

  e.g. $5^4/5^3 = 5^{(4-3)}$ *Must be the same base number*

- $(z^x)^y = z^{(x*y)}$

  e.g. $(3^2)^3 = 3^6 = 729$

Exponents: Rules

- $z^{-x} = 1/z^x$
- e.g. $3^{-2} = 1/3^2 = 1/9$
- $z^{½} = \surd z$ e.g. $4^{½} = \surd 4 = \pm 2$
- $z^1 = z$ e.g. $7^1 = 7$
- $z^0 = 1$ e.g. $8^0 = 1$, $219^0 = 1$

Exponents: Exercises

$2^4 * 2^2 = 2^?$

$(5^4)^2 = 5^?$

$\frac{7^3 * 3^3}{7^4 * 3^2} = ?$

$14^0 = ?$

Compound Interest

Let ***x*** be the amount of money you want to deposit at an annual interest rate ***r***.

After 1 year : $x + x^* r = x(1+r)$ $\quad$ $= x(1+r)^1$

After 2 years : $x(1+r) + r^*x(1+r)$ $\quad$ $= x(1+r)^*(1+r)$

$= x(1+r)^2$

After 3 years : $x(1+r)^2 + r^*x(1+r)^2$ $\quad$ $= x(1+r)^{2*}(1+r)$

$= x(1+r)^3$

So if ***t*** is the number of years you plan to invest for:

Future Value = Deposit * $(1 + r)^t$

Compound Interest

$$\textbf{Future Value} = \textbf{Deposit} * (\mathbf{1} + \mathbf{r})^{t}$$

If you deposit $10,000 compounded annually at a rate of 3%, how much will you have in 2 years?

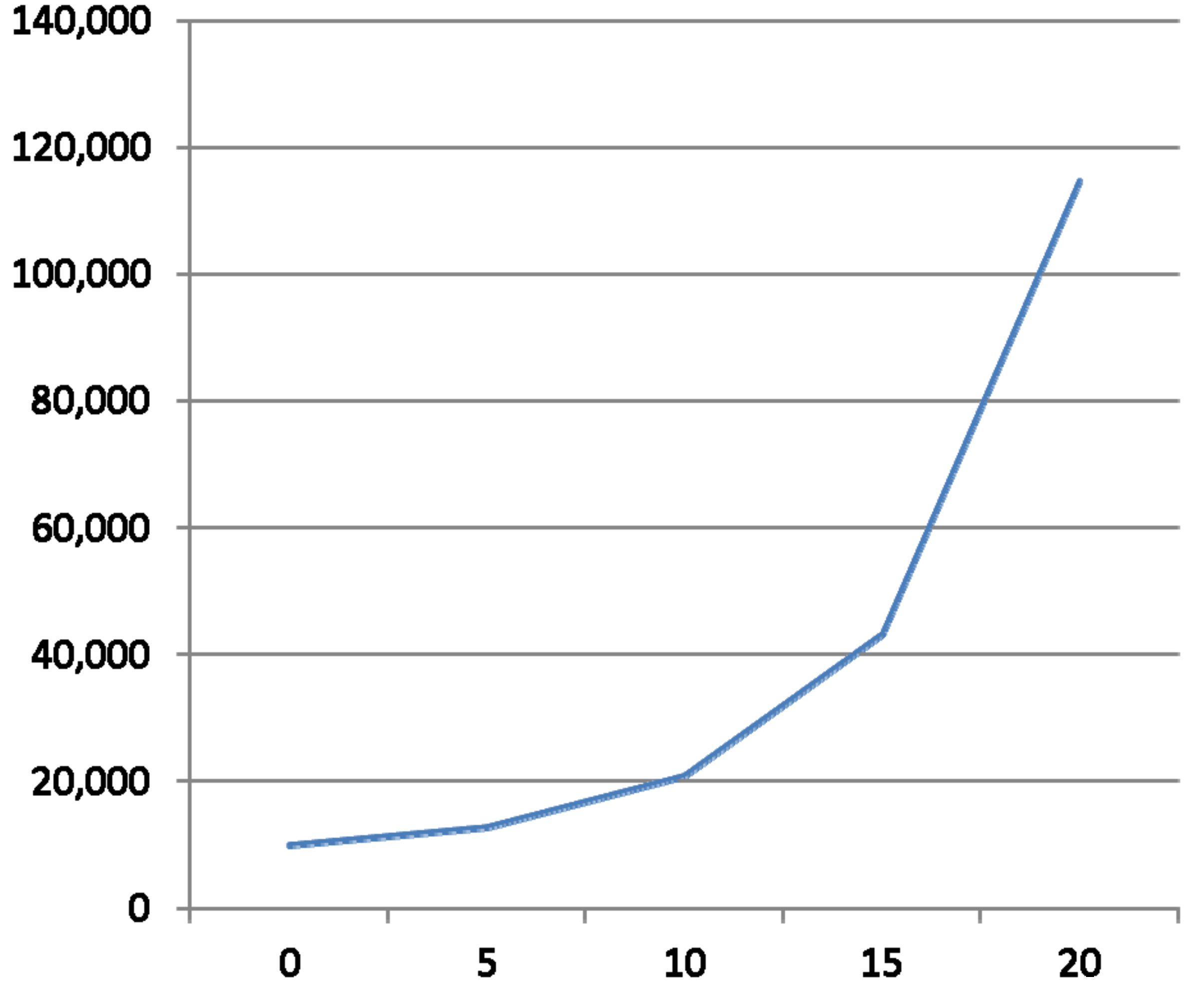
$$\text{Value in 2 years} = (10000) * (1 + 0.03)^2$$
$$= ?$$

How about in 25 years?

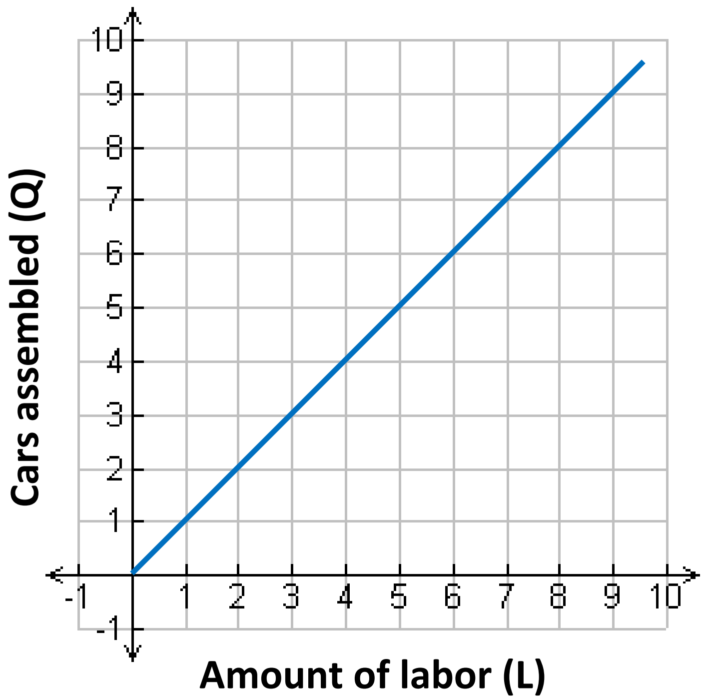
Exponential growth

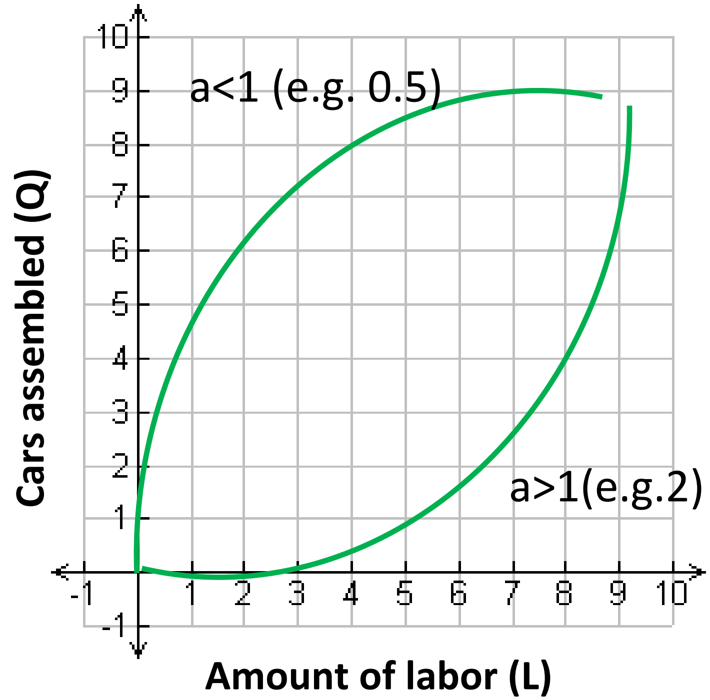
$f(t) = 1000 * (1 + 0.05)^t$

x	f(x)
0	$10,000
5	$12,762.82
10	$20,789.28
15	$43,219.42
20	$114,674.00



Exponents





Linear relationship:

$y = mx + b$

Nonlinear relationship:

$y = mx^a + b$

Non-linear Functions

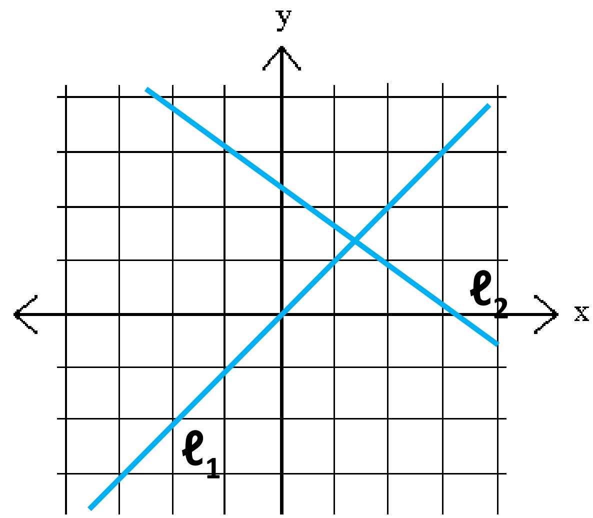
Algebraic Functions

- **Linear functions:**

  slope-intercept form f(x) = mx + b

  f(x) = 3x + 7 (or y = 3x + 7)

  Degree = 1 (no squared+ x)

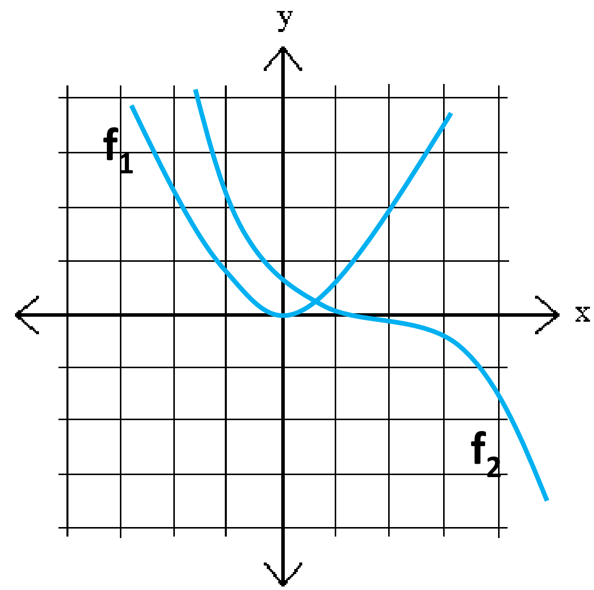


- **Nonlinear functions:** includes squared x, cubed x, etc.

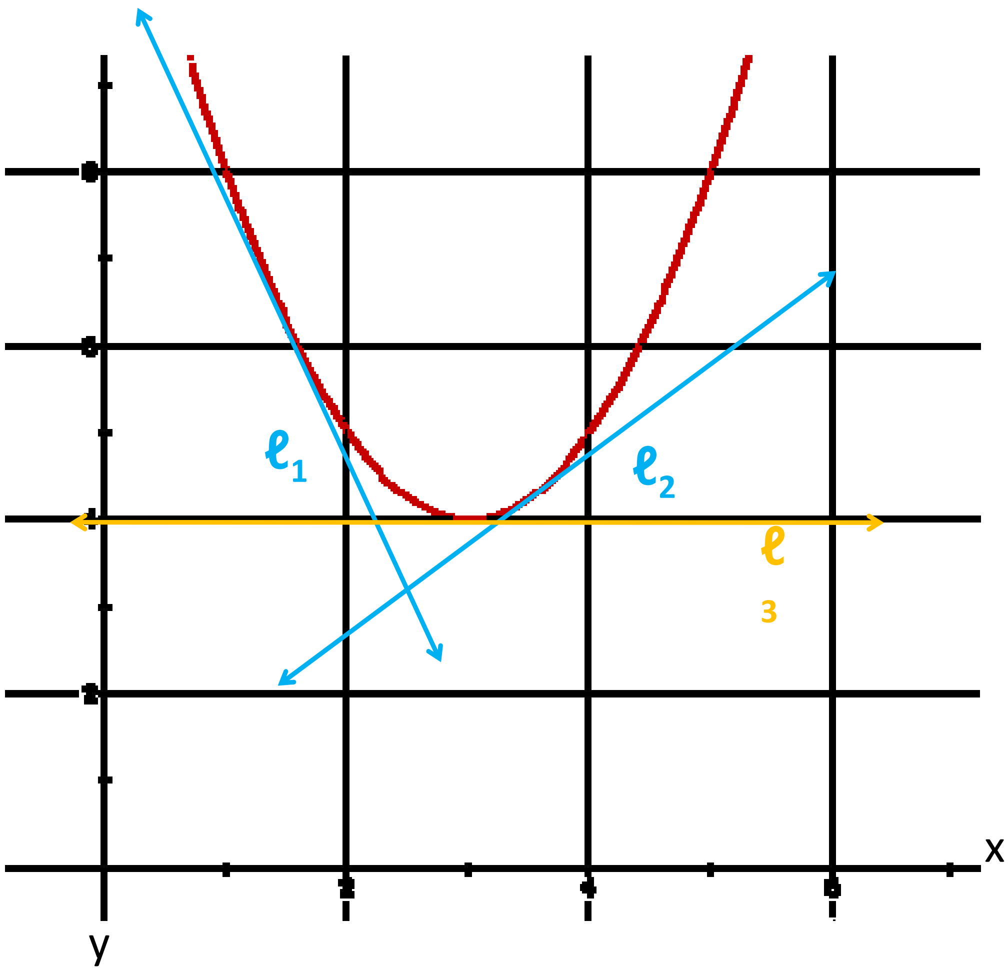
  Degree > 1

  $f(x) = x^2$

  $g(x) = -7x^3 + 5$



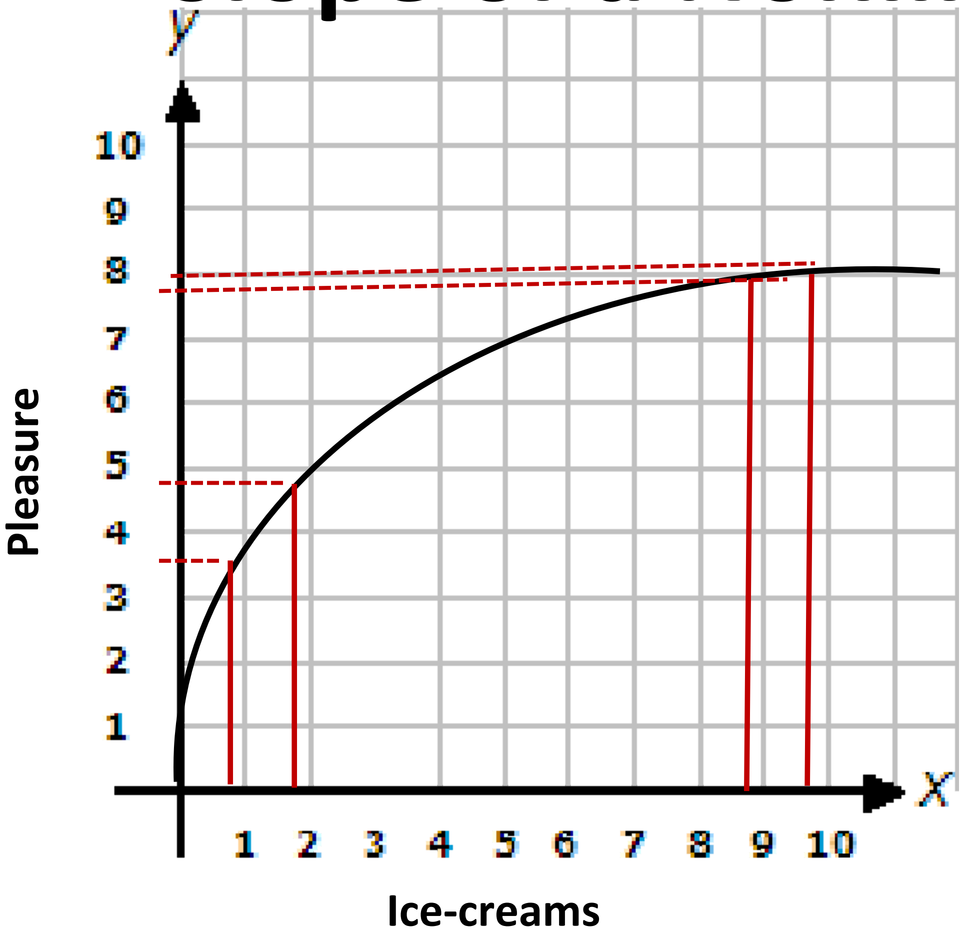
Slope of a Nonlinear Function



ℓ_1 : negative, steep slope
ℓ_2 : positive, less steep slope

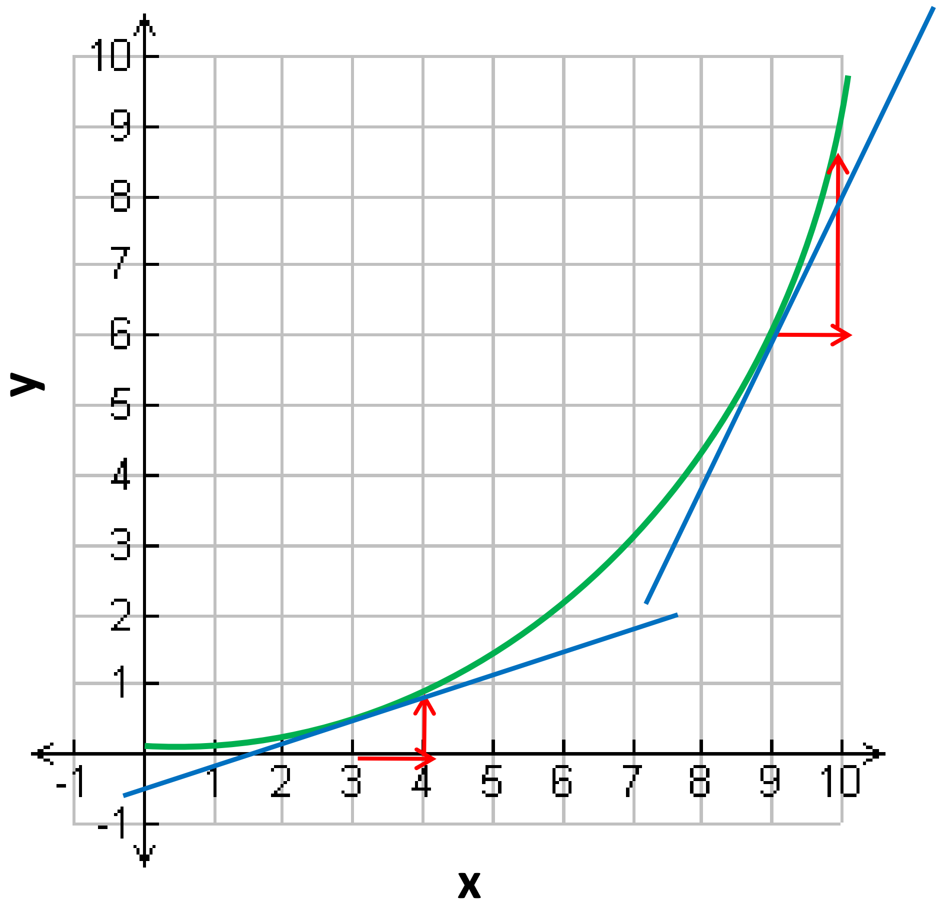
What is special about line

Slope of a Nonlinear function



What does the shape of this function suggest about the relationship between *x* and *y*?

Slope of Non-linear Functions



$Y = mx^a + b$

Different slopes along the curve.

Can draw lines at these points to see the different slopes.

Wednesday, January 15th

6 to 8pm

Math Camp Day 2

will cover

Calculus and Derivatives

and

Intro to Probability