Math Camp: Day 2

School of Public Policy
George Mason University
January 15, 2014
6:00 to 8:00 pm

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Course Outline

Monday 8/19

- Self-assessment test
- Online Modules

- Review of concepts, notation
- Algebra Review
- Linear functions
- Solving linear systems
- Coordinate geometry (JK)
- Exponents
- Non-linear functions

- Wrap-up, assignments

Wednesday 8/21

- **Review assignment**

- Intro to Derivatives
- Intro to Optimization
- Intro to Probability (Kirk)

- Post-test
- Course assessment

Assignment Review

The answers are online

Did you consider any of the questions particularly difficult?

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- **Intro to Derivatives**
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An example of a problem

Assume a firm making yo-yos has a production function with the shape $q=2K^{0.5}L^{0.6}$ (where q=output, K=capital services, and L=labor services, all per unit of time).

What are the returns to scale for this firm's technology?
Are there diminishing returns to either factor?

Taken from John Earle "Sample midterm exam II with sketched answers Fall 2012"

Why Derivatives?

- Derivatives are about **change**

- How fast is a car going?
- How fast is a car accelerating?

Physics

- How much it costs to produce more shoes?
- How much the cost changes when you produce more shoe?

Economics

Why Derivatives?

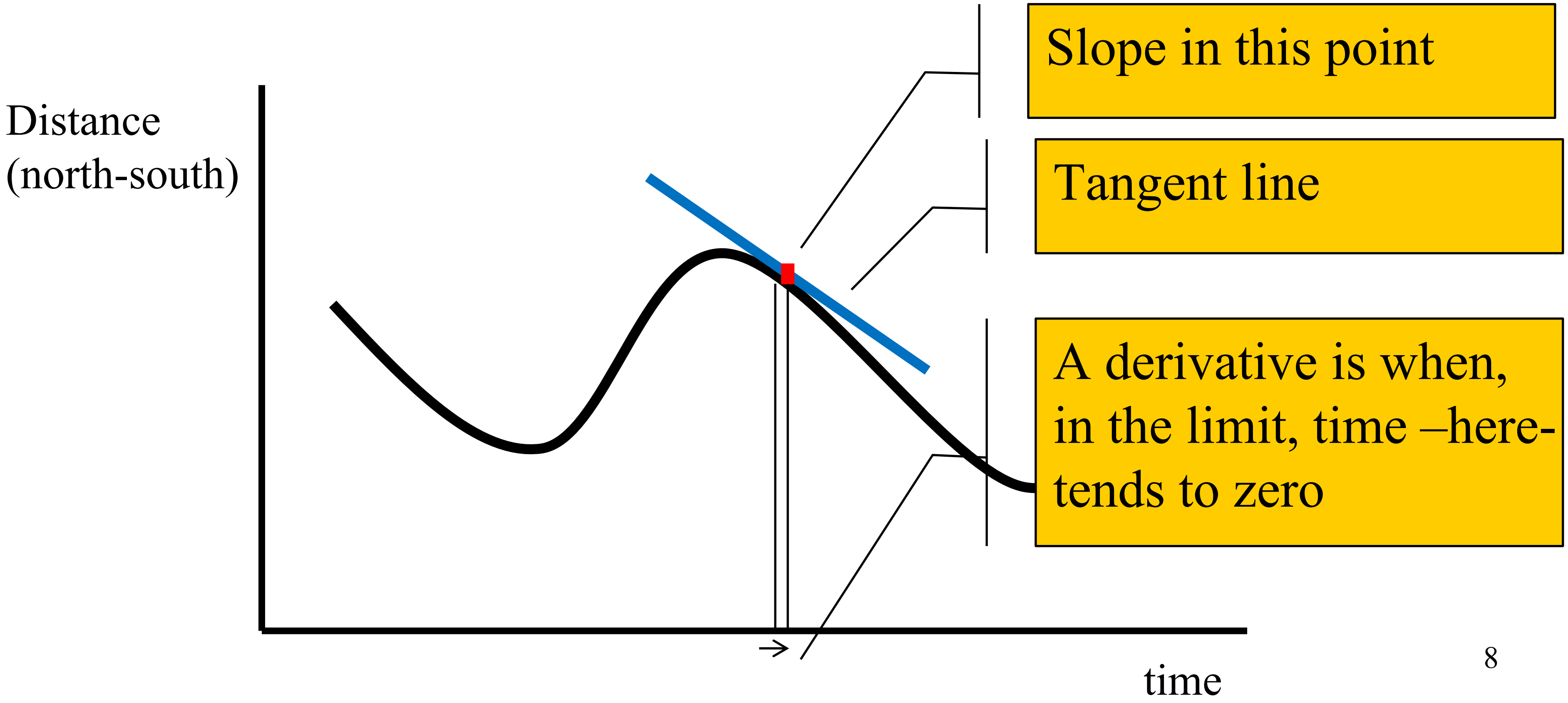
- More accurately, derivatives are about **limits**

- How fast is a car going –**right now**?
- How fast is a car accelerating –**right now**?

- How much it costs to produce **one** more shoe?
- How much the cost changes when
  you produce **one** more shoe?

Visually this is a derivative

A derivative is the **slope** of the tangent line "touching" the point of interest in the function

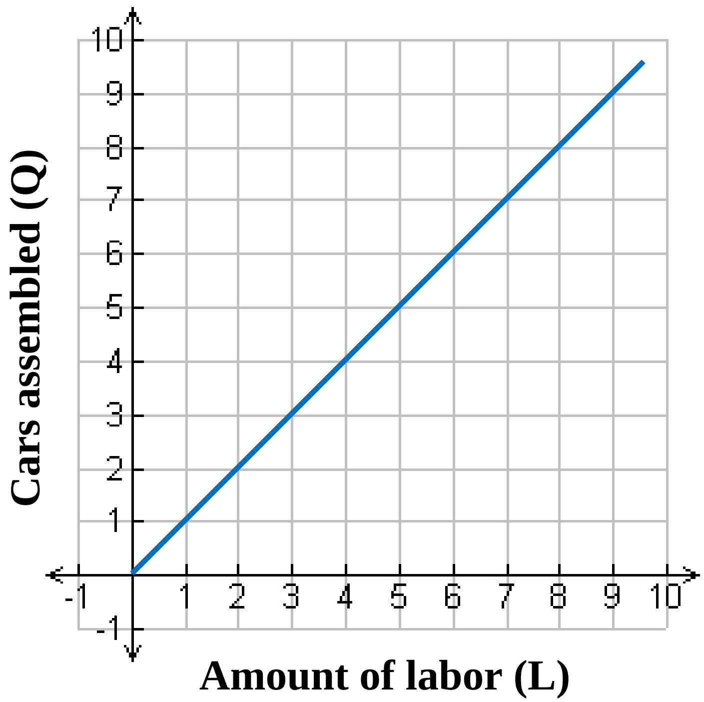


Why Derivatives in Economics?

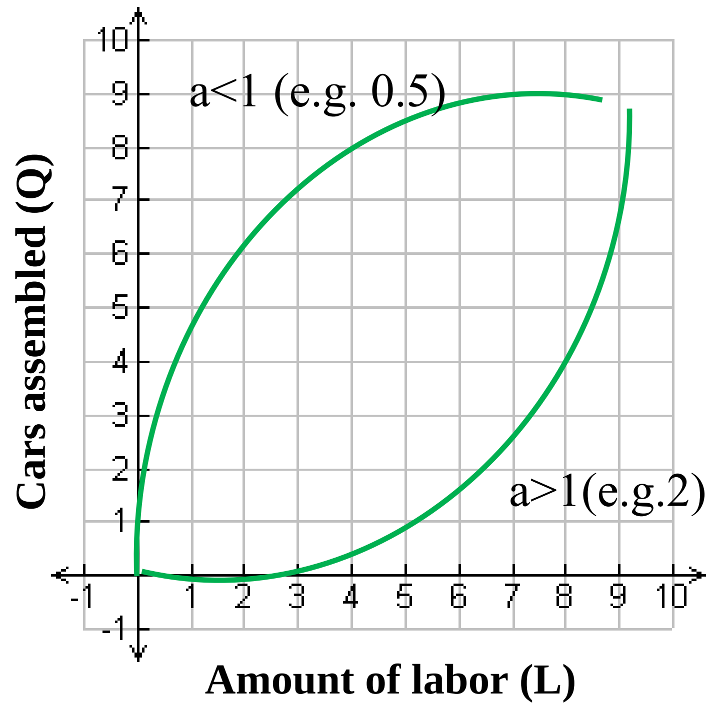
- Economic decision-making is about comparing the value of **the next unit** of something with its cost
- A decision that compares the costs and benefits of a small increase in the level of an activity is called a decision **"at the margin."**

- From a mathematical standpoint, **slope** is the concept that expresses the notion of "at the margin."

OK.. They are useful but some slopes are not so easy to calculate



Linear relationship:
$y = mx + b$

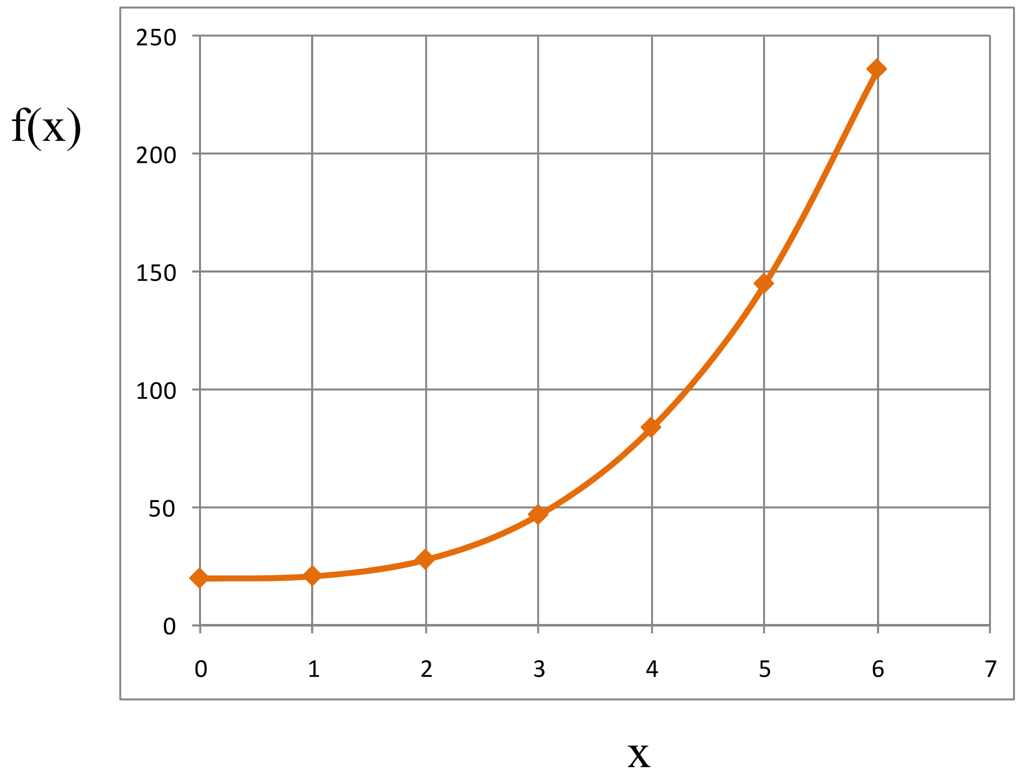


Nonlinear relationship:
$y = mx^a + b$

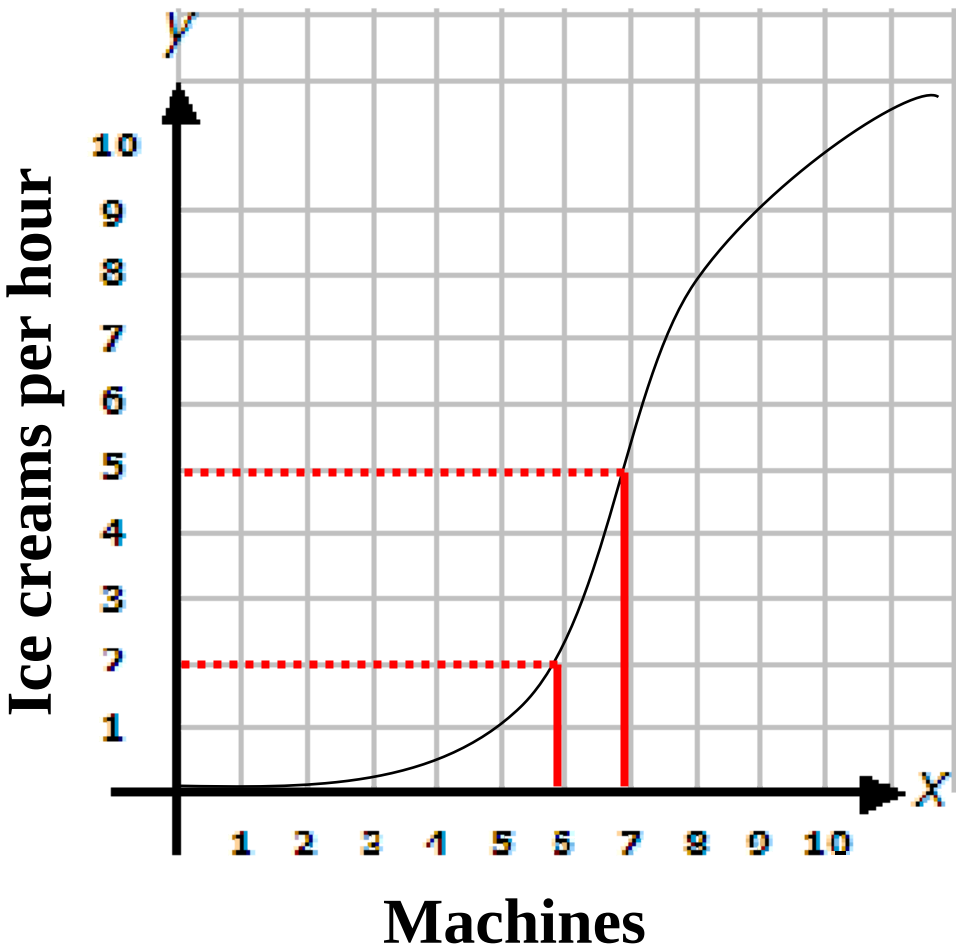
Moreover… slopes can change throughout the same function

$Y = f(x) = x^3 + 20$

x	Y= f(x)	Slope
0	20	0
1	21	3
2	28	12
3	47	27
4	84	48
5	145	75



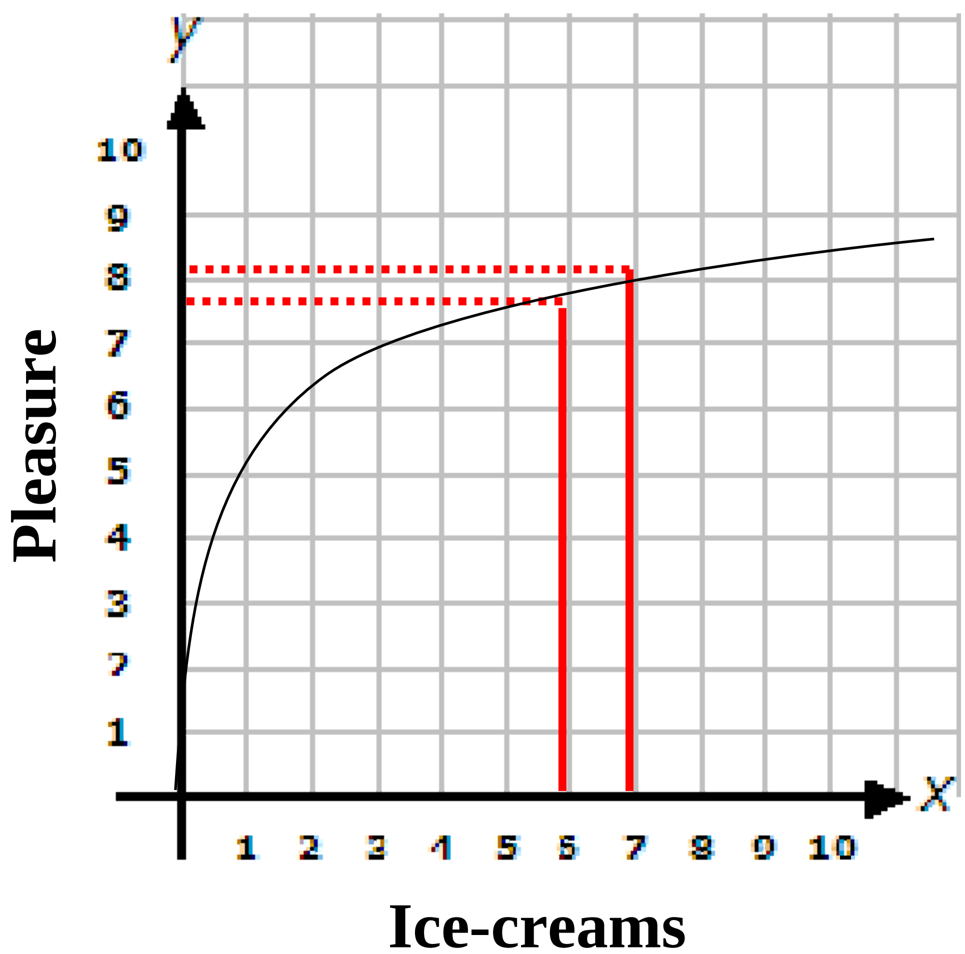
And how a slope change is important (I)



What does the shape of this function suggest about the relationship between *x* and *y*?

Increasing marginal product

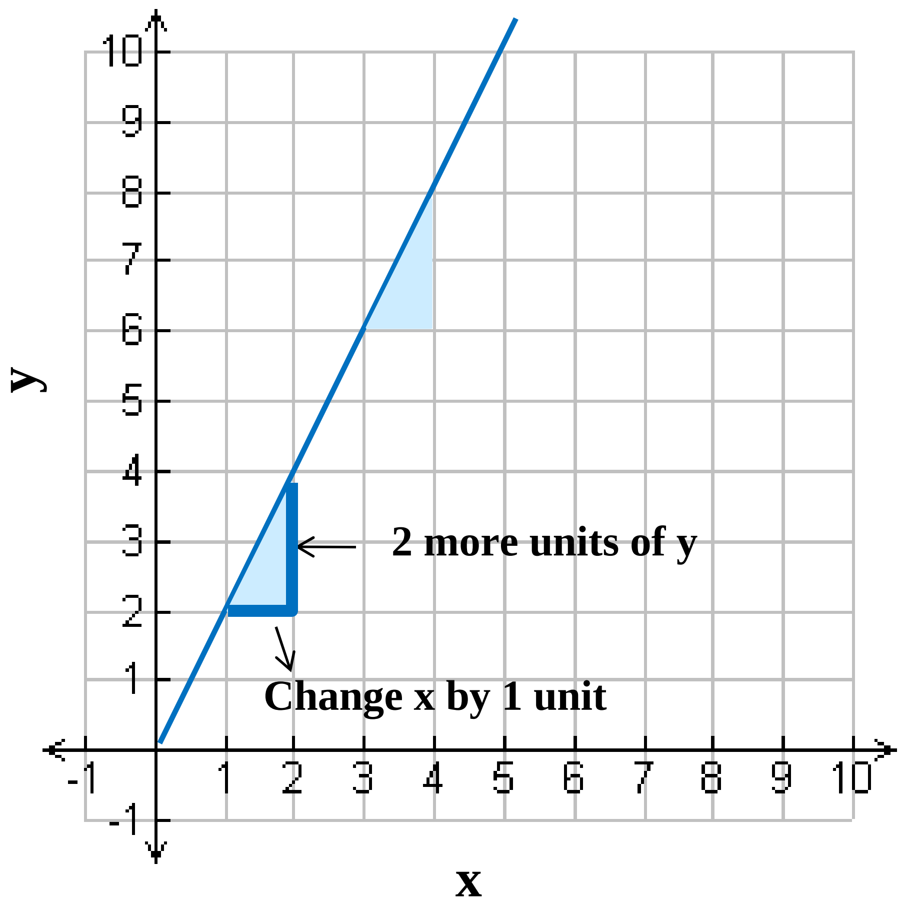
And how a slope change is important (II)



What does the shape of this function suggest about the relationship between *x* and *y*?

Decreasing marginal utility

Slope of Linear Functions



Y = mx + b

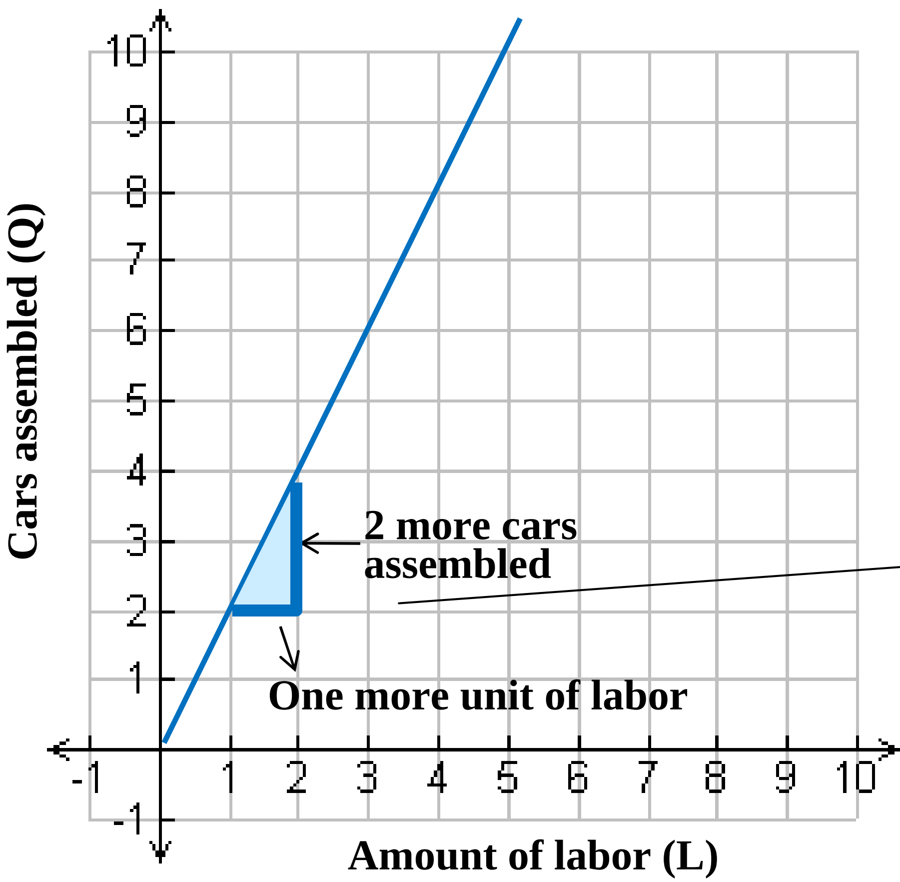
Slope = m = $\frac{\Delta y}{\Delta x}$ =

$\frac{\text{change in y}}{\text{change in x}} = \frac{2}{1} = 2$

Here, the slope is constant for all values of *x*

Derivative of a linear function is same as slope and remains constant

Decision at Margin: Example



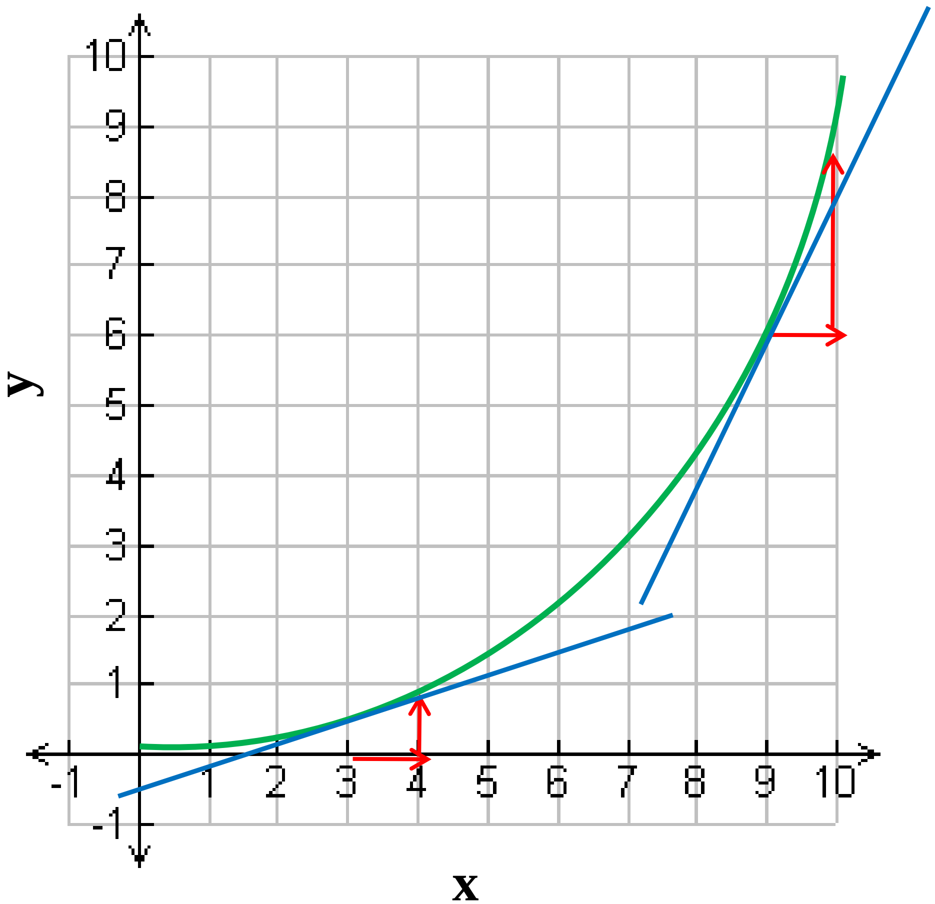
Marginal Productivity of Labor (MPL)

You're an owner of a car assembly plant. How much of an increase in car production can you expect if you increase labor by 1 unit?

Slope = MPL = Increase in the number of assembled cars achieved by hiring one more unit (=marginal increase) of labor.

$$y = mx + b \longrightarrow Q = 2L + 0$$

Slope of Non-linear Functions

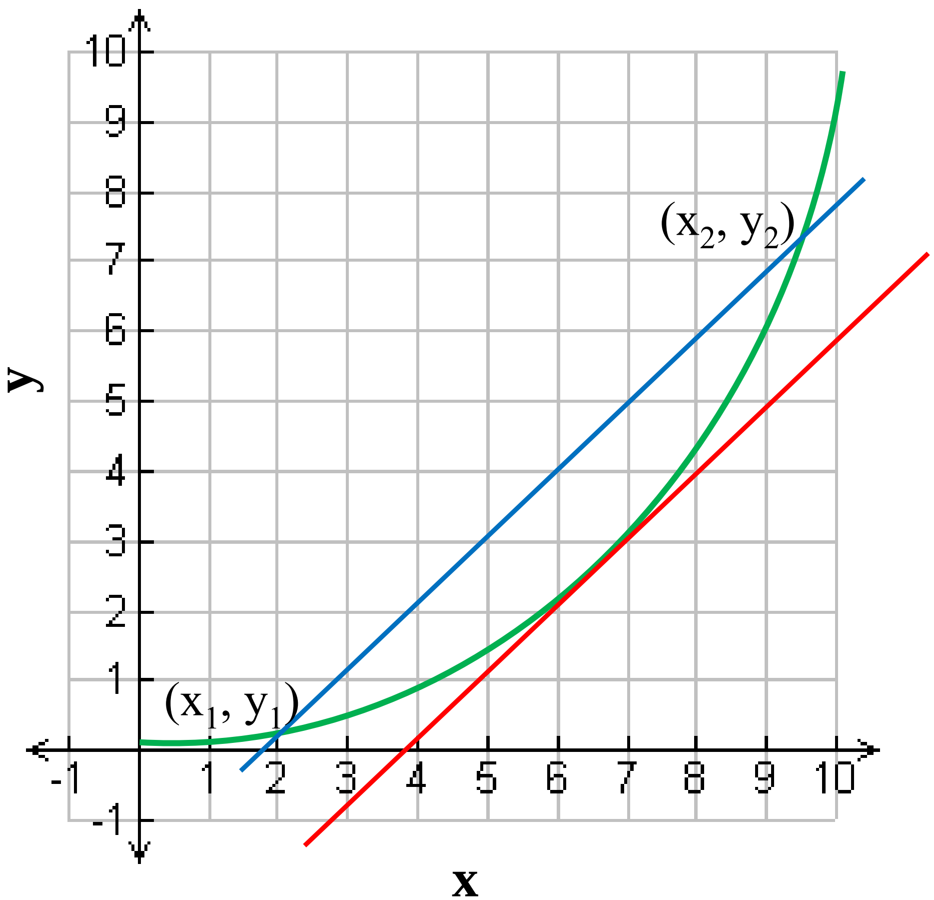


$Y = mx^a + b$

Different slopes along the curve.

Can draw lines at these points to see the different slopes.

Derivatives of Non-linear Functions



Slope is just

$$\frac{\text{change in y}}{\text{change in x}} = \frac{\Delta y}{\Delta x} \approx \frac{dy}{dx}$$

A derivative is just a slope where both the change in y and the change in x are restricted to being infinitesimally small.

Occurs where the line is tangent to the curve (touches at one point).

Is there an easy way to find the slope? Differentiation

If $y = f(x) = x^n$

Then
the derivative is $f'(x) = f'(x) = n*x^{n-1}$

e.g.
If the function is: $y = f(x) = x^3$

The derivative is $f'(x)= 3x^{3-1}$

$f'(x)= 3x^2$

Exercises

Take the derivative of the following functions:

$Y = v(t) = 8t$

$Y = f(x) = x^2$

$Y = \frac{3}{4} x^4$

$Y = \frac{1}{2} x^{0.5}$

Partial Derivatives: Introduction

$y = f(x)$ → derivative is the slope of the function at x

But what if the function we are using has more than one independent variable?

We can only evaluate one slope of each independent variable at a time

$y = f(x,z)$ → *partial* derivative is the slope along **one** input holding the other constant

We need to treat the input being held constant as a constant!

Partial Derivatives: Examples

Take the derivative of U with respect to "*x*"
(i.e. treat "z" as a constant)

$U(x,z) = x^3z$

$U_x\,(x,z) = \frac{\partial U}{\partial x} = 3x^2z$

Take the derivative of V with respect to "*z*"

$V(x,y) = 4x^3y^2$

$V_y\,(x,y) = \frac{\partial U}{\partial x} = 4x^3 * 2y$

$= 8x^3y$

Partial Derivatives: Exercises

Take the derivative with respect to x of the following functions

$U(x,y,z) = x^2y^3+2z$

$V(x,y,z) = \frac{1}{2} x^{0.5}y+2xz$

Partial Derivatives: Exercises

Take the derivative with respect to y of the following functions

$U(x,y) = x^2y^3$

$V(x,y) = ½ x^{0.5}y$

Example Functions with Two Inputs

Cobb-Douglas Production Function:

$Y = AL^{\alpha}K^{\beta}$

Utility Function:

$U = X^{\alpha}Y^{\beta}$

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An example of a problem

Suppose a competitive, profit-maximizing firm selling consultancy reports to government agencies has cost (in \$1000s) $C(q) = 10 + 5q + q^2$, where q is the number of reports it sells.

If it gets paid $P = 15$ (also in \$1000s) for delivering each report, how many reports will it produce and sell?

How many if government cutbacks result in P falling to 11?

What if further cuts reduce P to 7? For each of these questions, be sure to distinguish this firm's short and long run decisions.

Taken from John Earle "Sample midterm exam II with sketched answers Fall 2012"

Optimization

- This is why we are here.
- You will use this **constantly** in microeconomics.
- It is a powerful idea: by simplifying behavior you can generate predictions on how individuals, groups and firms will behave.

Optimization

- Optimization is about **maximization & minimization**

    - **Utility (=Satisfaction) maximization**: A consumer wants to maximize utility finding the best consumption basket (given a particular income level)

    - **Profit maximization**: A firm wants to find the most profitable supply level given demand for the product/services and the price of input(s)

    - **Cost minimization**. A firm wants to minimize the production cost finding the best mix of inputs (given a particular output level).

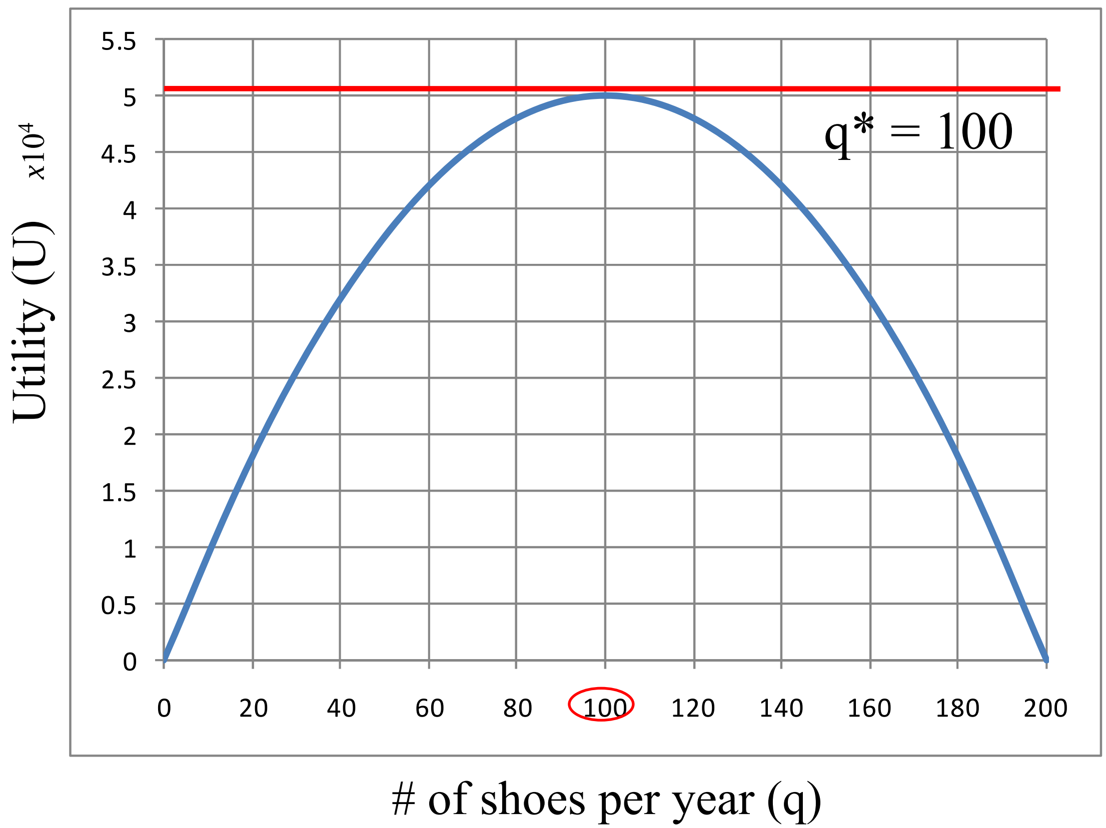
Optimization: let's think

- What is the maximum (y) value of a streight line?
- What is the maximum (y) value of a quadratic function?
- What is the minimum (y) value of a quadratic function?

Optimization: Let's buy some shoes

Utility/satisfaction *# shoes/yr*

Utility function: $U = f(q) = 1000q - 5q^2$



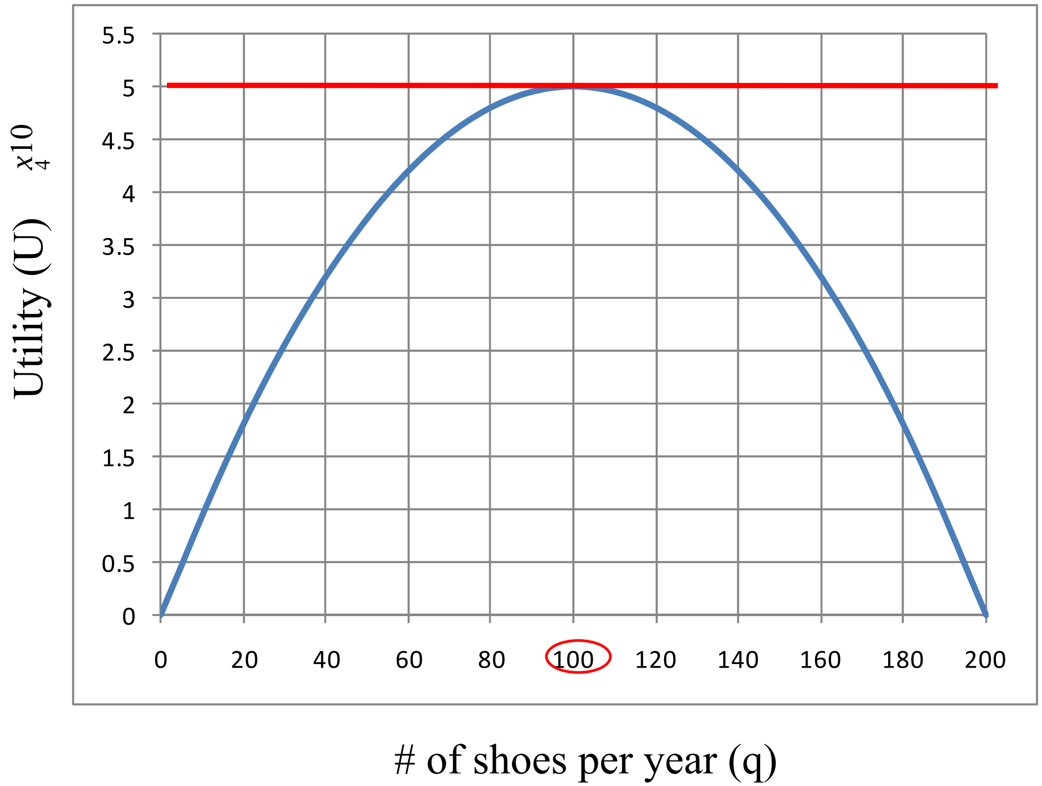
Where is utility maximized? How many shoes?

Slope at maximum point = derivative = 0

Set $U'(q) = 0$ to find the value of q that maximizes utility.

Optimization: Example

Utility function: $U = f(q) = 1000q - 5q^2$



$$U'(q) = \frac{df}{dq} = 1000\text{-}10q \boxed{= 0}$$

Solve for q*

$10q = 1000$

$q^* = 100$

Optimization: Examples

Find the optimal level of x* in the following functions:

$y = 8x^2 - 48x + 2$

$$\frac{dy}{dx} = \begin{aligned} &16x - 48 = 0 \\ &16x = 48 \\ &\mathbf{x = 3} \end{aligned}$$

$y = x^3 - 27x + 6$

$$\frac{dy}{dx} = \begin{aligned} &3x^2 - 27 = 0 \\ &3x^2 = 27 \\ &x^2 = 9 \\ &\mathbf{x = \pm 3} \end{aligned}$$

Optimization :Exercises

Find the optimal level of x* in the following function:

$$y = 8 - 3x + x^2$$

$$y = 50x - 2x^2$$

How do we interpret these values of x*?

Optimization

Find the optimal level of x* in the following functions:

$C(q) = 10 + 5q + q^2$

How do we interpret these values of x*?

Optimization

What if instead of assuming we are interested in a slope = 0 we are looking for a different slope?

We would be optimizing but subject to a restriction

Solve using slope=15, 11 and 7

Solve using Price=2q

$C(q) = 10 + 5q + q^2$

How do we interpret these values of x^*?

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An example of a problem

Suppose you drove to Georgetown on a Saturday night (first mistake) and looking for a place to park see a marginally illegal spot that you judge would have 20% chance of getting you a ticket if you take advantage of it, rather than paying the $30 cost of parking at the lot just down the hill. How do you decide whether to park illegally or park in the lot (assuming cruising around some more is not an alternative)?

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Kirk and Probabilities

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Post-test and Evaluation

Complete post-test → 15 minutes

Complete evaluation

Have a great semester!

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