

Loki's Practice Sets for PUBP555: Math Camp Spring 2014

Contents

Module 1.....	3
Rounding Numbers	3
Square Roots.....	3
Working with Fractions.....	3
Percentages.....	3
Order of Operations.....	4
Module 2.....	5
Algebraic Expressions.....	5
Geometry and Measurement	5
Module 3.....	6
Exponents.....	6
Roots and Radicals	6
Polynomials.....	7
Module 4.....	8
Solving Linear Equations with Equalities	8
With Inequalities.....	8
Introduction to Statistics.....	8
Module 5.....	9
Graphs of Linear Equations.....	9
Graphing Lines	10
Slopes and Intercepts.....	13
Equations of a Line.....	15
Module 6.....	18
Systems of Equations	18
Solving Systems of Equations using Graphical Intuition	20
Solving Systems of Equations using Substitution.....	21

Solving Systems of Equations using Elimination	21
Systems with three variables	22
Module 7	23
Functions.....	23
Recognizing Functions.....	24
Brief Intro to Conic Sections	25
Module 8	27
Derivatives using Power Rule.....	27
Derivatives using Product Rule	27
Derivatives using Chain Rule	27
Partial Derivatives	28
Optimization	28

Module 1

Rounding Numbers

1. Round to the nearest hundredth place 3.142857
2. Round to the second decimal place 2.71828
3. Round to the nearest tenth place 148.35
4. Round to tens 1,238
5. Round to hundreds 1,328,367

Square Roots

Estimate the positive square root or the following. For non-perfect squares estimate value of the two consecutive integers between which the value of the square root lies. Example: $2 < \sqrt{5} < 3$

1. $\sqrt{49} = ?$
2. $\sqrt{81} = ?$
3. $\sqrt{11} = ?$
4. $\sqrt{32} = ?$
5. $\sqrt{48} = ?$

Working with Fractions

Report your answers by rounding to the closest tenth digit or one decimal. Example: $\frac{1}{4} = 0.25$ or 0.3

1. $\frac{1}{3} + \frac{1}{3} = ?$
2. $\frac{13}{2} - \frac{12}{6} = ?$
3. $\frac{9}{4} \times \frac{8}{9} = ?$
4. $\frac{17}{5} \div \frac{9}{15} = ?$
5. $\frac{13}{0} = ?$

Percentages

Solve the following problems and where applicable round your answers to the nearest hundredth place:

1. If Adam earns \$3,200 a month and pays \$1,400/month in rent. What proportion of his monthly paycheck goes towards paying rent? What would this proportion be as a percentage?
2. What percent is 117 students of 500 students?
3. In 2000 the population of the Commonwealth of Virginia was 7,078,515. In 2010 the population increased to 8,001,024 people. What was the population percent change from 2000 to 2010 in the Commonwealth of Virginia?
4. Original price of shoes is \$125. There are two deals offered. (1) 20% off original price, then 25% off markdown price. (2) 40% off the original price. Which is the better deal?
5. Peter has an extensive collection of ties. He has 5 red ties, 3 blue ties, 2 green, 7 yellow, 2 purple, 8 black, and one silver colored tie. What percent of his ties are yellow?

Order of Operations

Simplify the following:

1. $13 \times (1 + 2) = ?$

2. $\frac{13}{3} - \frac{2}{6} + \frac{(32 - 30)}{2} = ?$

3. $(5 + (3 - 2 \times 5)) \times 4 = ?$

4. $\{ \{ \{ (200 \times 2) / 50 \} \times 5 \times 3 \times 50 \} / 25 \} \times 75 = ?$

5. $[(200 \times 2) / 50] \times 7500 + [(200 \times 2) / 50] \times 2 \times 3 \times 5000 + \{ \{ \{ (200 \times 2) / 50 \} \times 5 \times 3 \times 50 \} / 25 \} \times 75 = ?$

Module 2

Algebraic Expressions

Evaluate the following expression when $\Omega = 8$:

1. Ω^2
2. $3\Omega - \Omega/4 + 2\Omega^2$
3. $2\Omega + 1/8 * \Omega$
4. $((\Omega * 5 * 3 * 50)/25) * 75$
5. $\Omega^3 + 3\Omega^2 - 12\Omega + 4$

Geometry and Measurement

1. In a rectangular room one side is 80 feet long and the other is 40 feet long. What is the area of the room?
2. If your computer has a square screen and the length of its diagonal is 15 inches. What is the length of its side? What is its area? What is the length of its perimeter?
3. A circle has a radius of 4 centimeters. What is its perimeter? What is its area?
4. A right triangle has a hypotenuse of 5 inches and one side of 3 inches. What is the length of its third side?
5. You want to manufacture a cylindrical drink can of height 4 inches and a volume of 50.5 cubic-inches. What should its diameter be?

Module 3

Exponents

Simplify the following:

1. $\frac{3a^3 - 2a^3 + (b - b)}{3b^2 - b^2 - b^2} = ?$

2. $\frac{(\alpha^3 \beta^3)^{-1}}{(\alpha^{-3} \beta^{-3})} = ?$

3. $\frac{(f^3 g)^2}{(f g^{-3})^{-2}} = ?$

4. $\frac{(p^{-4})^{-3}}{(p^{-3} q^5)^{-5}} = ?$

5. $\frac{(\mu^{-42})^{(7-7)}}{(\beta^{-3} \Omega^5)^{(1.34-1.34)}} = ?$

Roots and Radicals

Simplify the following expressions to the format $x * \sqrt{y}$. Example: $\sqrt{50} = \sqrt{2*25} = 5\sqrt{2}$

1. $\sqrt{72} = ?$

2. $\sqrt{49} = ?$

3. $\sqrt{48} = ?$

4. $\sqrt{24} - \sqrt{6} = ?$

5. $\sqrt{63} + \sqrt{28} + \sqrt{112} = ?$

Solve for x:

1. $\sqrt{x} = 4$

2. $x^2 = 16$

3. $(1 + x)^2 = 25$

4. $1 + 2x + x^2 = 16$

5. $3 + 6x + 3x^2 = 48$

Polynomials

Expand and simplify the following expressions:

1. $(-7x^2 + 6x^4) - (3x^4 - 5x^7) = ?$

2. $(a + b) * (a + b) = ?$

3. $(x + y)^2 * (x + y) = ?$

4. $(x + ay) * (x + by) = ?$

5. Simplify by factoring: $x^2 - 3xy - 4y^2 = ?$

6. Use completing the square method to find the value of x. You are given that a, b, and c are all constants and x is the only variable.

$$ax^2 + bx + c = 0$$

Therefore $x = ?$

Module 4

Solving Linear Equations with Equalities

Solve for α in the following equations and wherever applicable round to one decimal:

1. $7\alpha - 12 = 3\alpha + 3$

2. $2\alpha + 4 = \alpha + 5$

3. $11\alpha - 10 = 10\alpha + 3$

4. $2\alpha - 4 = 8 - \alpha$

5. $\frac{1}{2}\alpha - 1 = \frac{\alpha}{3} + 5$

With Inequalities

Solve for β in the following equations:

1. $7\beta < 14$

2. $13\beta \leq 11$

3. $-6\beta + 12 < -14 + 3\beta$

4. $3\beta + 1 \geq -2 - \beta$

5. $4\beta + 4 \geq 16 - \beta^2$

Introduction to Statistics

1. Find the mean and the median of the following numbers: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

2. What is the median of the following numbers? 4, 5, 6, 13, 11, 2, 9

3. What is mode of the following numbers? 4, 13, 4, 5, 2, 13, 1, 6, 8, 9, 13, 13, 12, 5, 9, 6

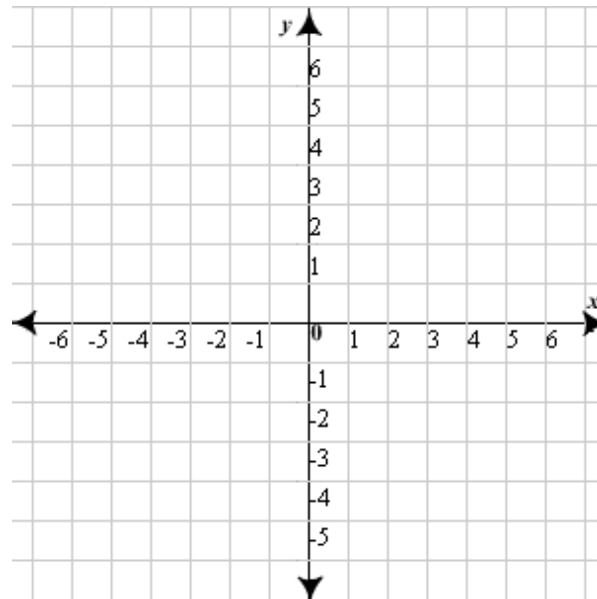
4. What is the mean, median and mode of the following numbers? 5,4,3,4,1,2,2

5. In a class there are 8 students who scored 85, 64, 98, 83, 78, 95, 77, 100 points on their exam. What is the mean score of the class? What is the median score?

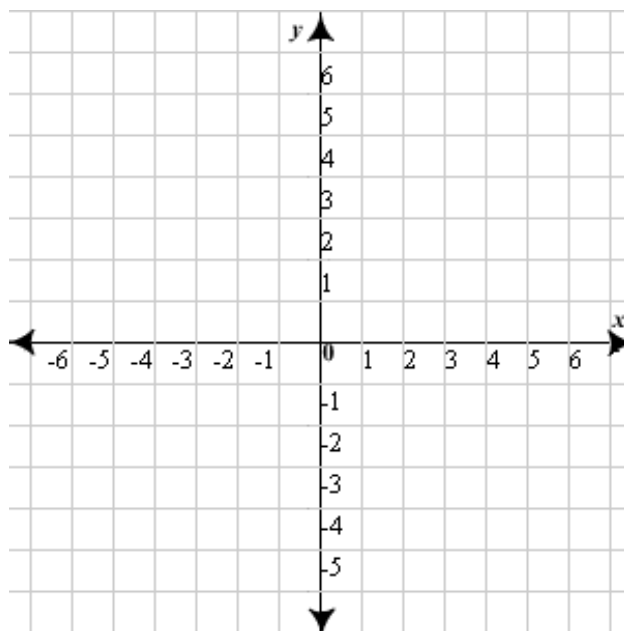
Module 5

Graphs of Linear Equations

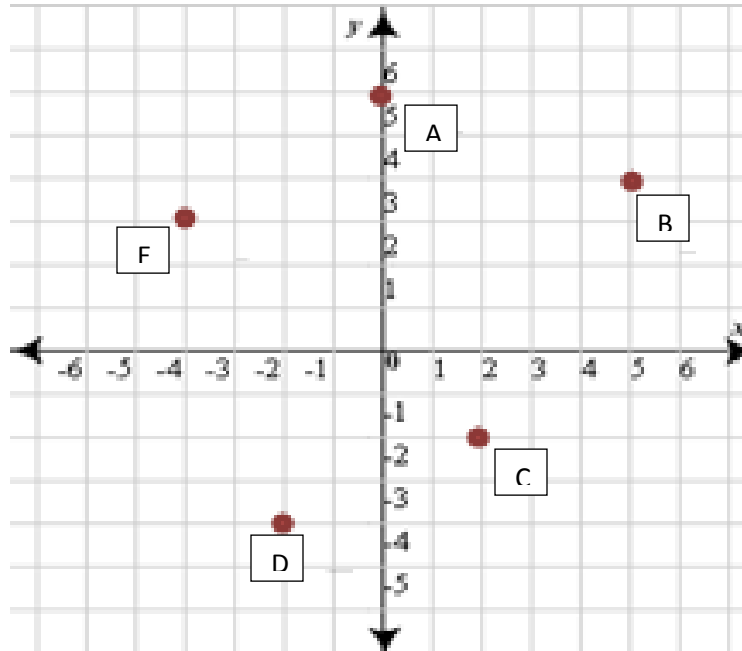
1. In which quadrant is the point $(4, -3)$



2. Graph the points $(-1, -4)$; $(3, 2)$; $(4, 0)$; $(-5, -6)$



2. Identify the following points from the graph:



4. What is the area of the triangle whose vertices are marked out by the ordered pairs $(4, 4)$; $(4, -4)$; and $(0, -4)$.

5. What is the length of the diagonal of a square that has its vertices as the ordered pairs $(-2, -2)$; $(2, 2)$; $(-2, 2)$; $(2, -2)$?

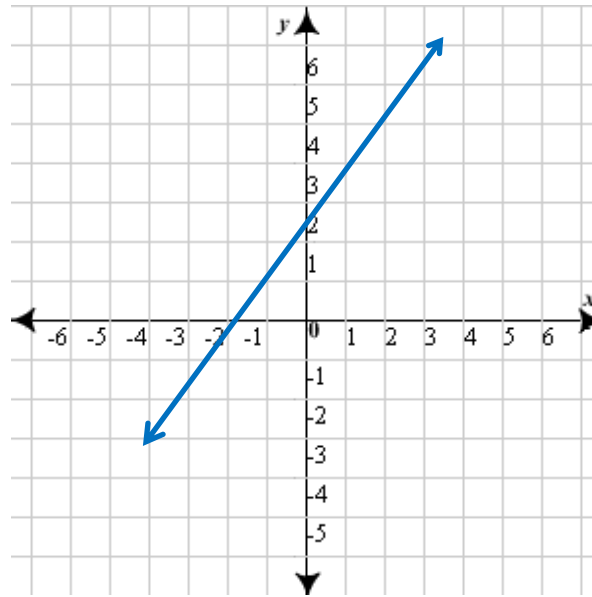
Graphing Lines

1. Which of the following ordered pairs is a solution to the equation $y = x + 4$?

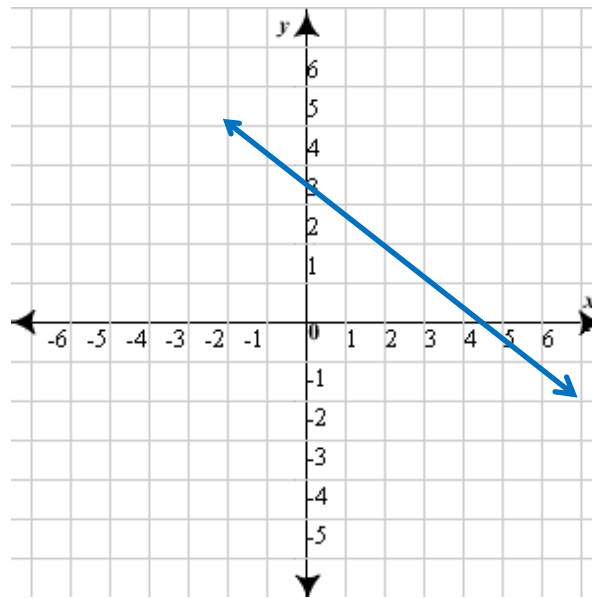
$(-2, 2)$ $(-1, 1)$ $(3, 5)$ $(4, 2)$ $(5, 9)$

2. Which of the following ordered pairs is a solution to the line graphed below?

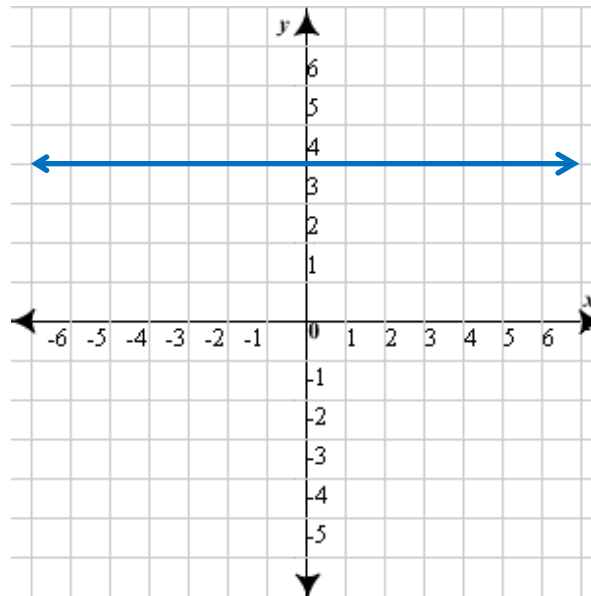
$(-2, 2)$ $(-1, 1)$ $(3, 5)$ $(4, 2)$ $(5, 3)$



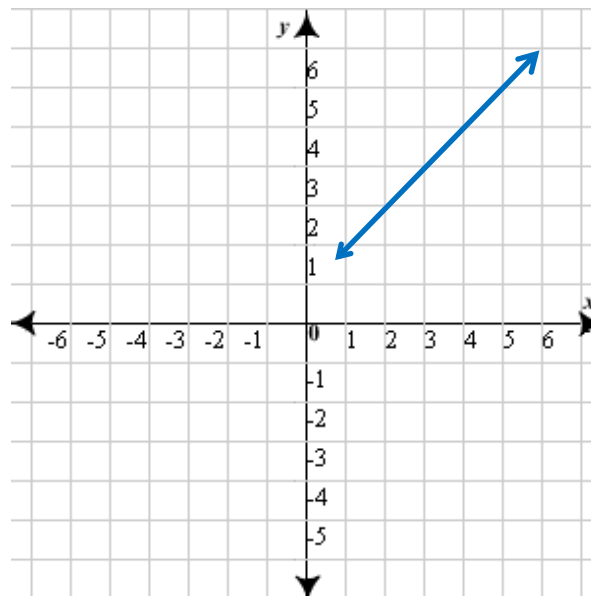
3. Based on the line graphed below, what is the value of y when $x = 2$?



4. Based on the line graphed below, what is the value of x when $y = 4$?

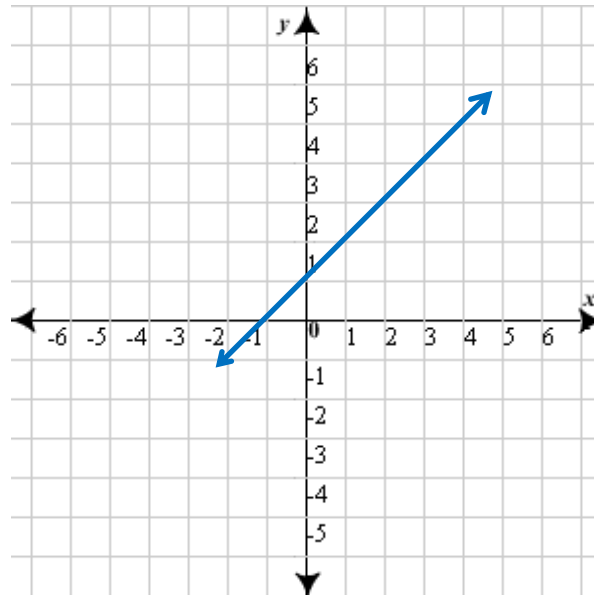


5. The following graph shows the relationship between price of bread (on y -axis) and consumption of bread (on x axis). What happens to consumption of bread as its price increases? Is it increasing, decreasing, or staying the same? Is the result what you would expect?

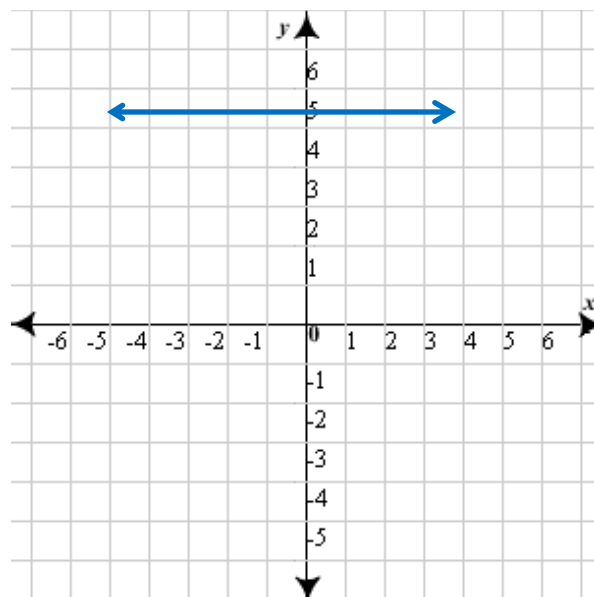


Slopes and Intercepts

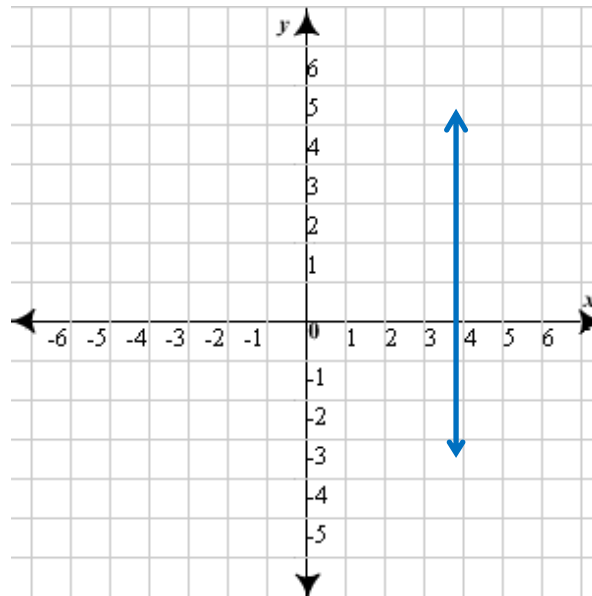
1. What is the slope of the line below? What is its y-intercept? What is its x-intercept?



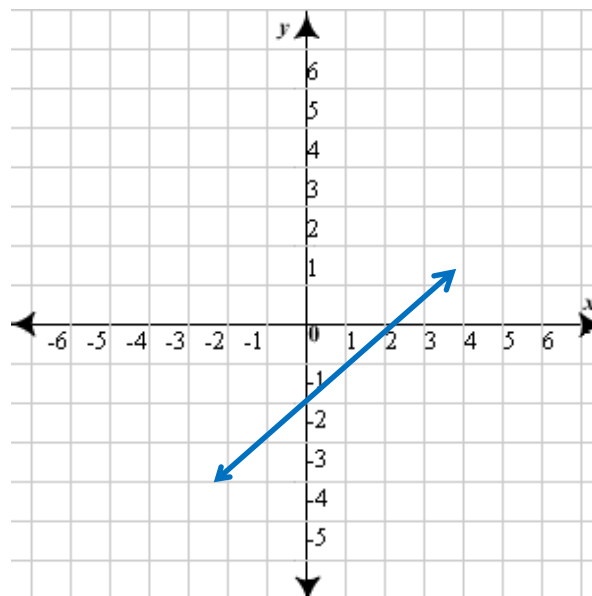
2. What is the slope of the line below? What is its y-intercept? What is its x-intercept?



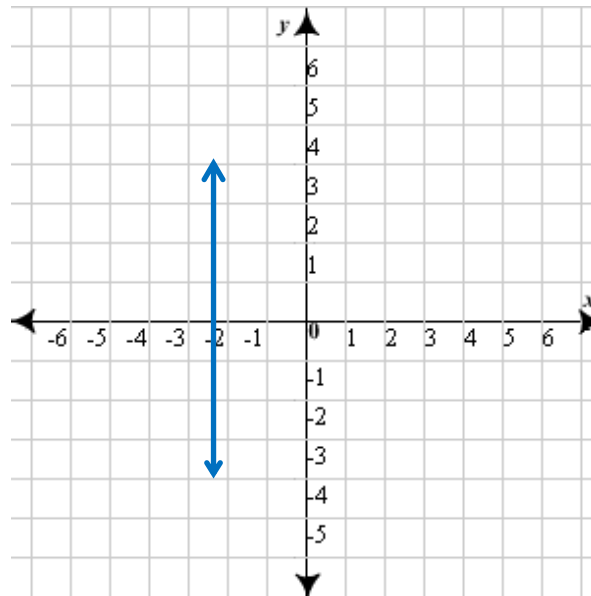
3. What is the slope of the line below? What is its y-intercept? What is its x-intercept?



4. Graph a line parallel to the given line and with a y-intercept at $(0,2)$?

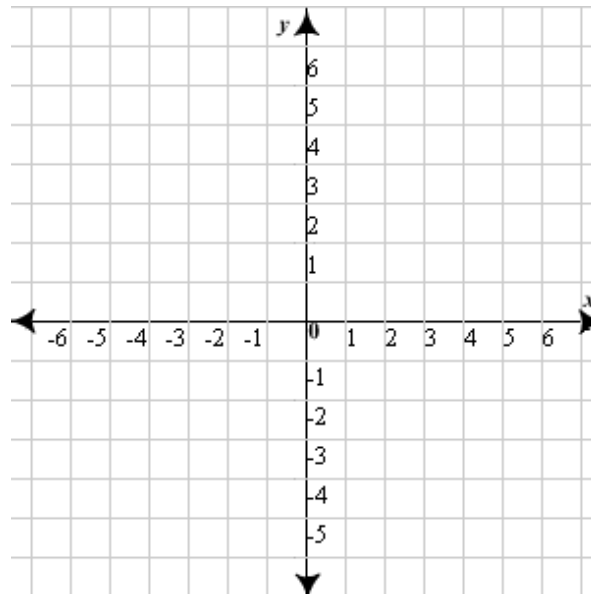


5. Graph a line parallel to the given line and with a x-intercept at (2,0)?

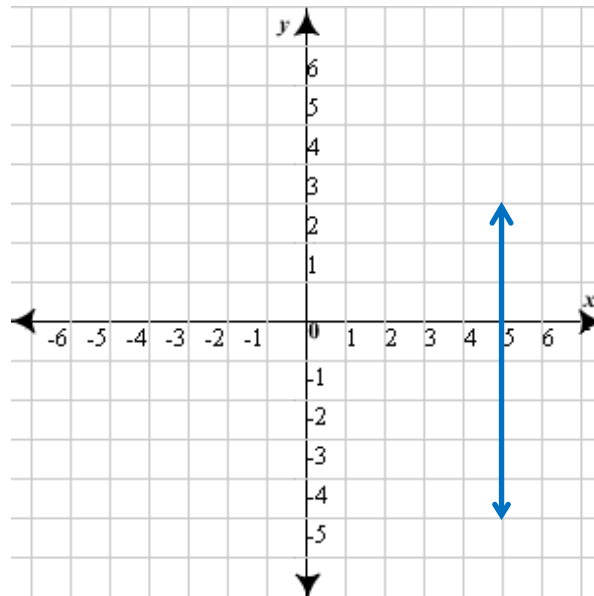


Equations of a Line

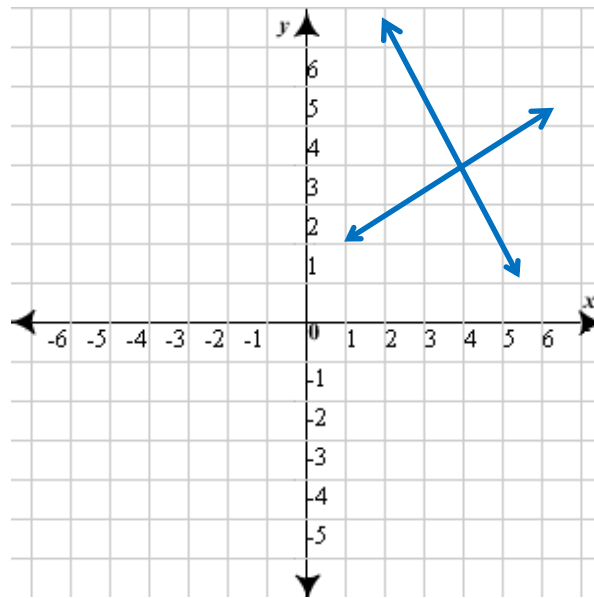
1. Graph a line whose equation is $3y = 12x + 6$



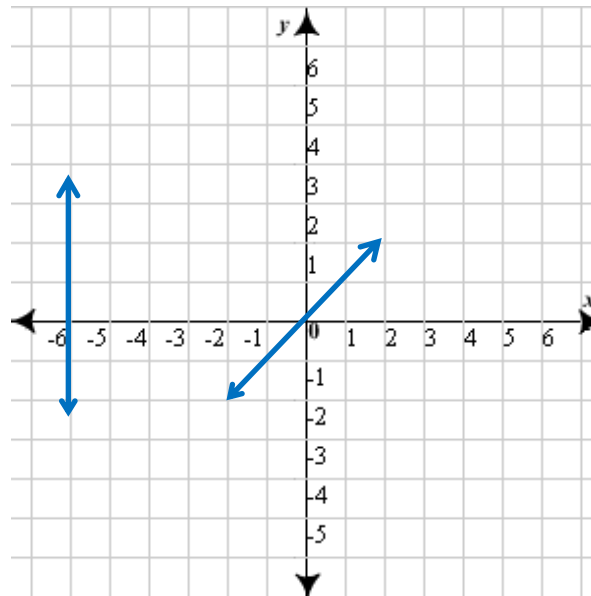
2. What is the equation of the line graphed below?



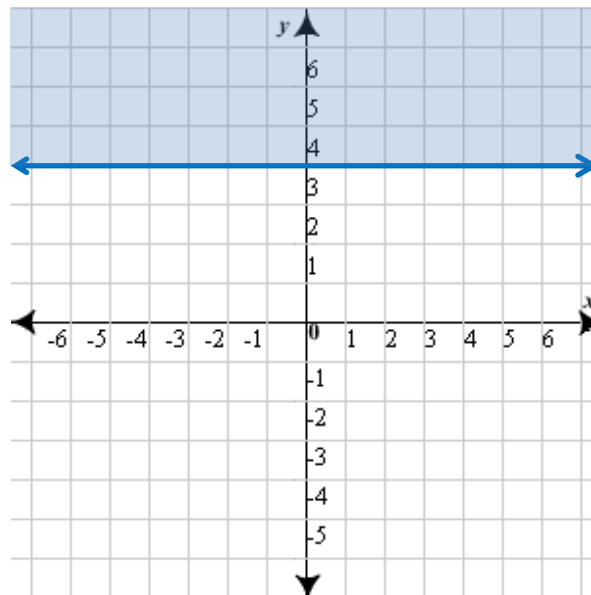
3. Name a point that satisfies the equations of both the lines below.



4. Name a point that satisfies the equations of both the lines below.



5. What is the inequality represented in the graph below?



1. Name 4 points that lie on the perimeter of a circle whose equation is $x^2 + y^2 = 16$

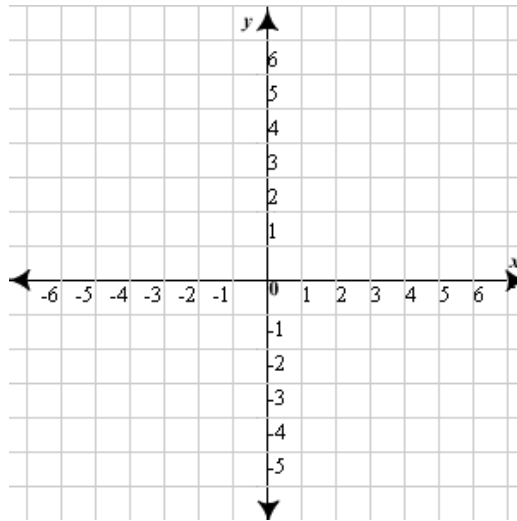
Module 6

Systems of Equations

1. Graph this system of equations and solve

(i) $y = 3x + 2$

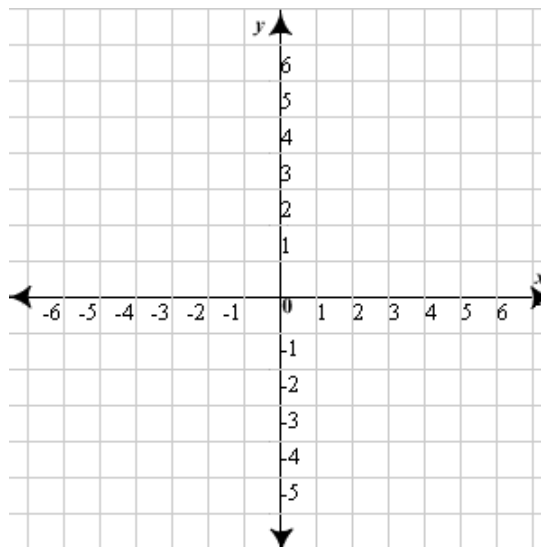
(ii) $y = x + 3$



2. Graph this system of equations and solve

(i) $2y = 4x + 2$

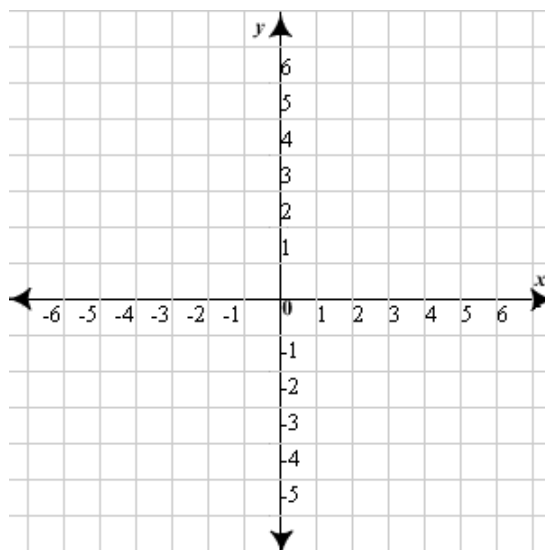
(ii) $y = 2x + 1$



3. Graph the lines represented by the following ordered pair and solve the system of equations

(i) $(0,2)$; $(4, 0)$

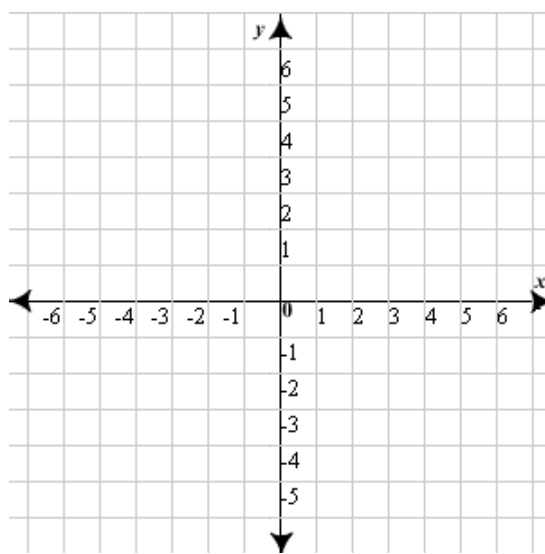
(ii) $(1,1)$; $(2,2)$



4. Graph the lines represented by the following ordered pair and solve they system of equations

(i) $(0,0)$; $(4, 0)$

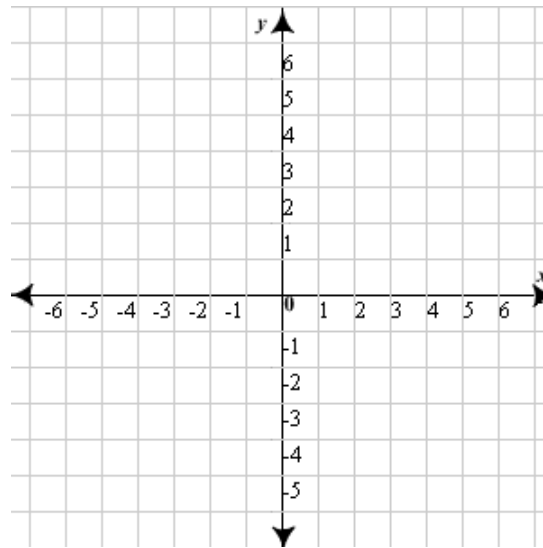
(ii) $(-2,-2)$; $(3, 3)$



5. Graph the lines represented by the following ordered pairs. Is there a solution for this system of equations?

(i) $(-4, 0)$; $(0, 4)$

(ii) $(0, -4)$; $(4, 0)$



Solving Systems of Equations using Graphical Intuition

1. How many solutions exist for this system of equations? (One solution; two solutions; infinite solutions; or, none)

(i) $15x - 3y = -27$

(ii) $y = -3 - 3x$

2. How many solutions exist for this system of equations? (One solution; two solutions; infinite solutions; or, none)

(i) $2y = -3x + 8$

(ii) $8y = -12x + 32$

3. How many solutions exist for this system of equations? (One solution; two solutions; infinite solutions; or, none)

(i) $2y = -3x + 8$

(ii) $8y = -12x + 80$

4. Fill in the blanks below to complete the equation of a line that shares infinitely many solutions with a line of equation

$$3x - 2y = 12$$

Ans: ___ x - ___ y = 6

5. Write the equation to a line that is parallel to the line $y = 5x + 3$ and passes through the point (0, 4)

Solving Systems of Equations using Substitution

1. (i) $2x + 5y = -4$ (ii) $y = -4x + 10$

2. (i) $-3x + 2y = 12$ (ii) $x = -3y + 7$

3. (i) $2x - 4y = -6$ (ii) $x = 5y - 6$

4. (i) $-5x - 3y = 7$ (ii) $y = -x + 1$

5. (i) $6x - 6y = 6$ (ii) $y = 5x - 9$

Solving Systems of Equations using Elimination

1. (i) $3x - 3y = 6$ (ii) $-3x - 5y = -46$

2. (i) $6x + y = 27$ (ii) $-5x - y = -23$

3. (i) $-2x + y = -5$ (ii) $-3x - y = -25$

4. (i) $-2x - 3y = -44$ (ii) $2x + 5y = 64$

5. (i) $-2x - 3y = -15$ (ii) $2x + 5y = 17$

Systems with three variables

1. (i) $3x - y + z = 2$ (ii) $2x + 2y - z = 12$ (iii) $x + y - z = 6$
2. (i) $x - 2y + 3z = 7$ (ii) $2x + y + z = 4$ (iii) $-3x + 2y - 2z = -10$
3. (i) $2x - 4y + 5z = -33$ (ii) $4x - y = -5$ (iii) $-2x + 2y - 3z = 19$
4. (i) $x + y + z = 2500$ (ii) $y - z = 6000$ (iii) $.6x + 0.7y + 0.8z = 162$
5. (i) $4x + y - 2z = 0$ (ii) $2x - 3y + 3z = 9$ (iii) $-6x - 2y + z = 0$

Module 7

Functions

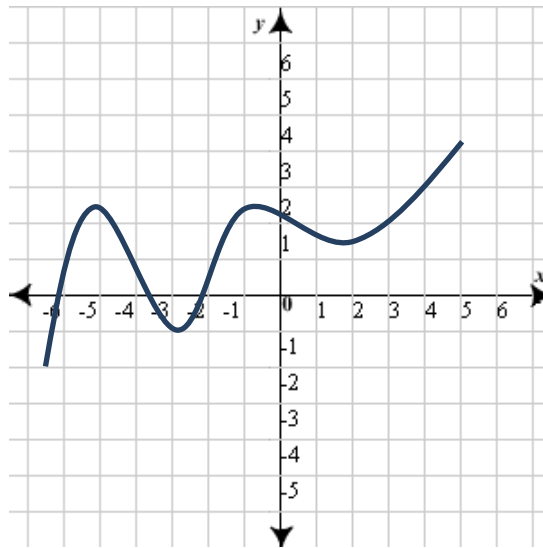
1. Write the equation of function that represents the relationship between cost of tuition and number of course credits taken, if each course credit costs \$345. What would be the cost of tuition if you registered for 5 course credits?

2. If you toss a ball from the top of founders hall building, its height (in feet) at any time (in seconds) is represented by the function $h(t) = 250 - 8t^2$. What is the height of the ball two seconds after you drop it?

3. Your bank is offering you a certificate of deposit (CD) option at 6% for 4 years. If you invest \$5,000 today, how much will you have in the CD when it matures? Formula for compound interest is:

Future Value = Present Value * $(1 + \text{interest rate})^{\text{time in years}}$, round your answer to the hundredth place.

4. In the graph below what is $f(4)$?

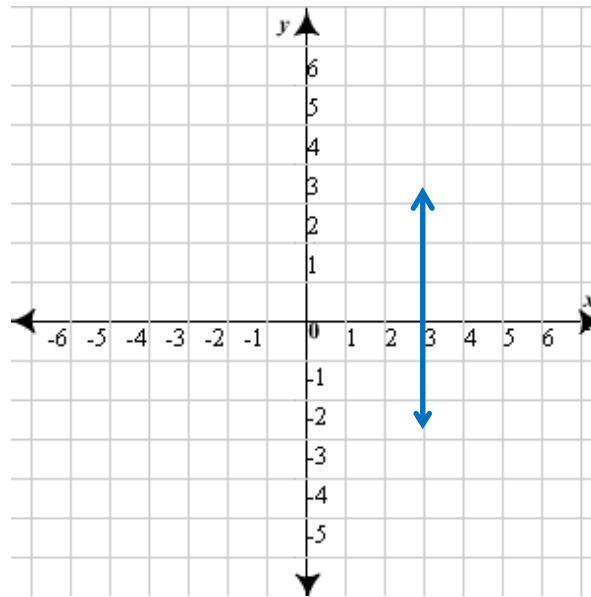


5. For $f(h) = | -3h + 10 |$ what is $f(5)$?

Recognizing Functions

Answer if the following relations represent a function or not.

1. The value of y is always 13 less than x .
2. $y = 3x + 4$
3. $x^2 + y^2 = 1$
4. $y^2 = x$
5. Is y a function of x in the following graph?



6. John has a very unique method of making his coffee. If he is making between 1 to 5 cups of coffee he will always put 5 tablespoons of coffee ground into his machine. However if he makes more than 5 cups, he adds one more table spoon of coffee per additional cup. Can the number cups of coffee he makes be represented as a function of the number of tablespoons of coffee grounds he used?

7. At your local grocer you can buy a candy bar for \$1; milk for \$2; cookies for \$0.50; and pencils for \$0.25. Can you represent your price as a function of the products bought?

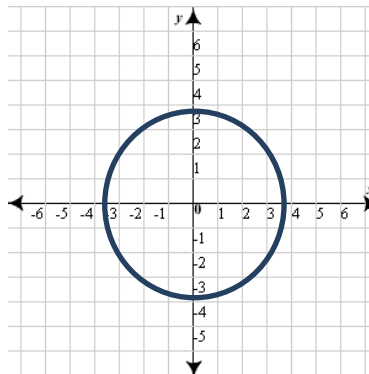
8. Given the table below, can you represent y as a function of x ?

y	x
0	15
5	12
10	4
15	15
20	12

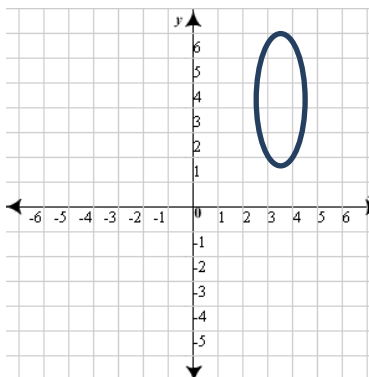
Brief Intro to Conic Sections

Identify the following conic sections: Is it circle? Hyperbola? Parabola? Or an Ellipse?

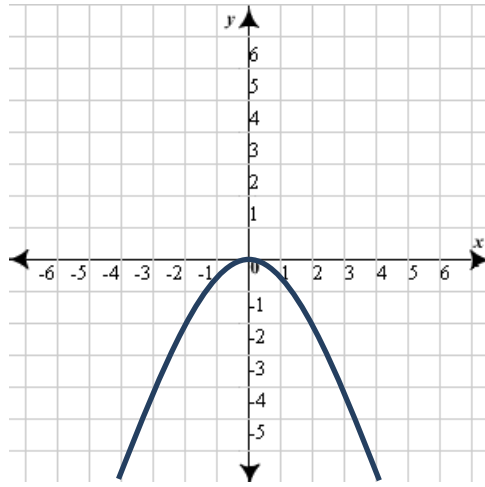
1.



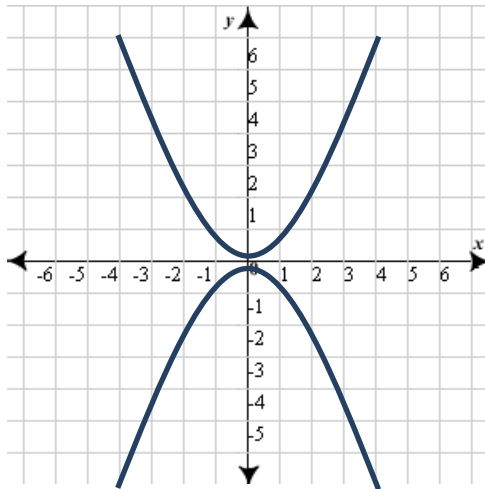
2.



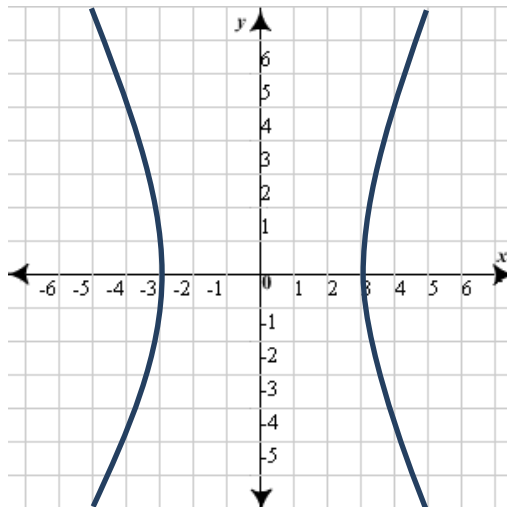
3.



4.



5.



Module 8

Derivatives using Power Rule

Take the first derivative of the following

1. $f(x) = 12x^8$

2. $f(x) = -13x^{-5}$

3. $f(x) = 1/x^3$

4. $f(x) = 12/x^4$

5. $f(x) = 11/7$

Derivatives using Product Rule

Take the first derivative of the following

1. $y = (x^3 + 2x + 1) * (2x + 2)$

2. $y = x^4 * (4x - 2)$

3. $y = (x^2 - 4x) * (1 + 3x^2)$

4. $y = (x - 2) * (x^3 - 2)$

5. $y = x^3 * (2x + 5)$

Derivatives using Chain Rule

1. $y = (2x + 3)^2$

2. $y = \sqrt{x^2 + 3x + 2}$

3. $y = (21 + 4x^2)^5$

4. $y = \frac{3x^2 - 4x}{x^3}$

5. $y = \frac{5x - 2x^2}{x^{-3}}$

Partial Derivatives

1. Take the first order partial derivative of $f(x,y)$ with respect to x for

$$f(x,y) = x^2 + 2xy + y^2$$

2. Take the first order partial derivative of $f(x,y)$ with respect to y for

$$f(x,y) = (xy^3 + 2x + 1)(2y + 2)$$

3. Take the **second order** partial derivative of $f(x,y)$ with respect to x for

$$f(x,y) = (x^3 - y^3)^5$$

4. Take the **second order** partial derivative of $f(x, y)$ with respect to y for

$$f(x,y) = x^2y + 2x + y$$

5. Take the **first order** partial derivative with of $f(x,y)$ with respect to x and the **second order** partial derivative of $f'(x,y)$ with respect to y for

$$f(x,y) = x^3y^3 + 3x^3y$$

Optimization

1. Maximize utility for the given utility function $U(q) = 1000q - q^2$

2. Find the critical point of the function $g(z) = z^2 + 4z + 3$

3. Find the critical point of the function $p(q) = q^3 - 3q^2 + q$

4. Find the critical point with respect to x for $U(x,y) = 20x + 80y - x^2 - 2y^2$

5. Find the critical point with respect to y for $U(x,y) = 20x + 80y - x^2 - 2y^2$