Loki's Practice Sets for
PUBP555: Math Camp Spring 2014

Contents

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Module 1

Rounding Numbers

1. Round to the nearest hundredth place 3.142857

   3.14

2. Round to the second decimal place 2.71828

   2.72

3. Round to the nearest tenth place 148.35

   148.4

4. Round to tens 1,238

   1,240

5. Round to hundreds 1,328,367

   1,328,400

Square Roots

Estimate the positive square root or the following. For non-perfect squares estimate value of the two consecutive integers between which the value of the square root lies. Example: $2 < \sqrt{5} < 3$

1. $\sqrt{49}$ = ?

   $\sqrt{49} = 7$

2. $\sqrt{81}$ = ?

   $\sqrt{81} = 9$

3. $\sqrt{11}$ = ?

   As $3^2 = 9$ and $4^2 = 16$; we know that $3 < \sqrt{11} < 4$

4. √32 = ?

As $5^2 = 25$ and $6^2 = 36$; we know that $5 < \sqrt{32} < 6$

5. √48 = ?

As $6^2 = 36$ and $7^2 = 49$; we know that $6 < \sqrt{48} < 7$

Working with Fractions

Report your answers by rounding to the closest tenth digit or one decimal. Example: 1/4 = 0.25 or 0.3

1. $\frac{1}{3} + \frac{1}{3} = ?$

$$\frac{1}{3} + \frac{1}{3} = \frac{(1+1)}{3} = \frac{2}{3} = 0.7$$

2. $\frac{13}{2} - \frac{12}{6} = ?$

$$\frac{13}{2} - \frac{12}{6} = \frac{13 \times 3}{2 \times 3} - \frac{12}{6} = \frac{39 - 12}{6} = \frac{27}{6} = 4.5$$

3. $\frac{9}{4} \times \frac{8}{9} = ?$

$$\frac{9}{4} \times \frac{8}{9} = \frac{9 \times 8}{4 \times 9} = \frac{{}^{1}\cancel{9} \times {}^{2}\cancel{8}}{{}_{1}\cancel{9} \times {}_{1}\cancel{4}} = 2.0$$

4. $\frac{17}{5} \div \frac{9}{15} = ?$

$$\frac{17}{5} \div \frac{9}{15} = \frac{17 \times {}^{3}\cancel{15}}{{}_{1}\cancel{5} \times {}_{3}\cancel{9}} = \frac{17}{3} = 5.7$$

5. $\frac{13}{0} = ?$

Division by zero is undefined

Percentages

Solve the following problems and where applicable round your answers to the nearest hundredth place:

1. If Adam earns $3,200 a month and pays $1,400/month in rent. What proportion of his monthly paycheck goes towards paying rent? What would this proportion be as a percentage?

Proportion of monthly paycheck is $\frac{1400}{3200} = \frac{14}{32} = \frac{7}{16}$

The percentage is 7/16 = 0.4375 or 43.75% of Adam's paycheck goes to rent.

2. What percent is 117 students of 500 students?

117/500 = 0.234 or 23.40% students

3. In 2000 the population of the Commonwealth of Virginia was 7,078,515. In 2010 the population increased to 8,001,024 people. What was the population percent change from 2000 to 2010 in the Commonwealth of Virginia?

Percent change = $\frac{\text{(New value–Old value)}}{\text{Old Value}} = \frac{(8{,}001{,}024-7{,}078{,}515)}{7{,}078{,}515} = \frac{922{,}509}{}$ = 0.1303 or 13.03%

4. Original price of shoes is $125. There are two deals offered. Which is the better deal?

(i) 20% off original price, then 25% off markdown price.

20% off $125 = $125 – ($125*0.20) = $125 – $25 = $100 = markdown price

Another 25% off markdown price = $100 – ($100 * 0.25) = $100 - $25 = $75

(ii) 40% off the original price.

40% off $125 = $125 – ($125*0.40) = $125 – $50 = $75

So both deals offer the same final discount.

5. Peter has an extensive collection of ties. He has 5 red ties, 3 blue ties, 2 green, 7 yellow, 2 purple, 8 black, and one silver colored tie. What percent of his ties are yellow?

Total number of ties = 5 red + 3 blue + 2 green + 7 yellow + 2 purple + 8 black + 1 silver = 28 ties

There are 7 yellow ties, therefore proportion of his ties that are yellow is 7/28 = 0.25, or 25% of Peter's ties are yellow.

Order of Operations

Simplify the following:

1. 13 x (1 + 2) = ?

= 13 x (3) = 39

2. $\frac{13}{3} - \frac{2}{6} + \frac{(32 - 30)}{2}$ =?

$= \frac{13}{3} - \frac{2}{6} + \frac{(2)}{2} \quad = \frac{13 \text{x} 2}{3 \text{x} 2} - \frac{2}{6} + \frac{1}{1} \quad = \frac{26 - 2}{6} + \frac{1}{1} \quad = \frac{24}{6} + \frac{1}{1} \quad = 4 + 1 \quad = 5$

3. (5 + (3 – 2 x 5)) X 4 = ?

= (5 + (3 – 10)) x 4 = (5 + (-7)) x 4 = (5 – 7) x 4 = -2 x 4 = -8

4. ({[(200*2)/50]* 5*3*50}/25)*75 = ?

= ({[400/50]* 5*3*50}/25)*75

= ({8* 5*3*50}/25)*75 = (6000 / 25)*75 = 6000*3 = 18000

5. [(200*2)/50] * 7500 + [(200*2)/50] * 2*3*5000 + ({[(200*2)/50]* 5*3*50}/25)*75 = ?

The trick is to simplify the expression. We see that [(200*2)/50] is a common expression between all the "+" signs. So we can factor it out to get:

[(200*2)/50] * **(**7500 + 2*3*5000 + ({5*3*50}/25)*75**)** Next we can solve inside the parenthesis to get:

[8] * **(**7500 + 30000 + 30*75**)** = [8] * **(**7500 + 30000 + 2250**)** = 8 * 39750 = 318,000

Module 2

Algebraic Expressions

Evaluate the following expression when Ω = 8:

1. Ω^2

    $8^2 = 64$

2. $3\Omega - \Omega/4 + 2\Omega^2$

    $3*8 – 8/4 + 2*8^2 = 24 – 2 + 2*64 = 24 – 2 + 128 = 150$

3. 2Ω + 1/8 * Ω

    2*8 + 1/8 * 8 = 2* 8 + 8/8 = 16 + 1 = 17

4. ((Ω * 5*3*50)/25)*75

    ((8 * 5 * 3 * 50)/25)*75 = (8 * 5 * 3 * 2) * 75 = 40 * 6 * 75 = 1800

5. $\Omega^3 + 3\Omega^2 - 12\Omega + 4$

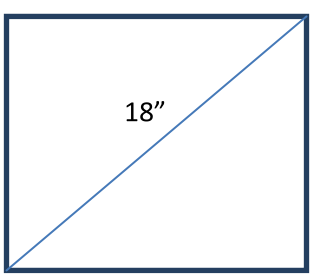
    $(8^3 + 3*8^2 – 12*8 +4) = 512 + 192 - 96 + 4 = 512 + 196 – 96 = 512 + 100 = 612$

Geometry and Measurement

1. In a rectangular room one side is 80 feet long and the other is 40 feet long. What is the area of the room?

Area of rectangle = length * width = 80 feet * 40 feet = 3,200 feet^2

2. If your computer has a square screen and the length of its diagonal is 18 inches. What is the length of its side? What is its area? What is the length of its perimeter?



<---------Side----------->

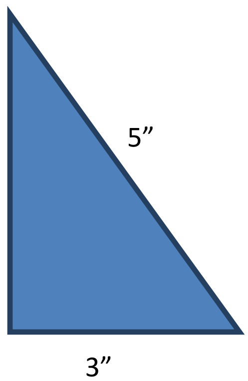
Pythagoras theorem: sum of the squares of the sides of a triangle is equal to the square of the hypotenuse
i.e.: $(\text{side 1})^2 + (\text{side 2})^2 = (\text{hypotenuse})^2$
Since we have a square screen with equal sides, the above expression can be simplified to:
$2 * (\text{side})^2 = (\text{hypotenuse})^2$
As we know hypotenuse = 18” we get;
$2 * (\text{side})^2 = 18^2$ then dividing both side by 2 we get
$(\text{side})^2 = 324 / 2 = 162$ and taking square root of both sides;
side = √162 = **12.73”**
Area of square = square of the side = **162 inches^2**
Perimeter of square = 4 * side = **648 inches.**

3. A circle has a radius of 4 centimeters. What is its perimeter? What is its area?

Perimeter of circle = 2*π*radius = 2 * π * 4 = 25.13 cm

Area of a circle = $2*\pi*(\text{radius})^2 = 2 * \pi * 4^2 = 2 * \pi * 16 = 100.53$ cm^2

4. A right triangle has a hypotenuse of 5 inches and one side of 3 inches. What is the length of its third side?



Pythagoras theorem: sum of the squares of the sides of a triangle is equal to the square of the hypotenuse

i.e.: $(\text{base})^2 + (\text{height})^2 = (\text{hypotenuse})^2$

We know the base = 3” and the hypotenuse = 5” so we have;

$(3)^2 + (\text{height})^2 = (5)^2$ OR

$9 + (\text{height})^2 = 25$ and subtracting 9 from both sides we get

$9 + (\text{height})^2 – 9 = 25 - 9$

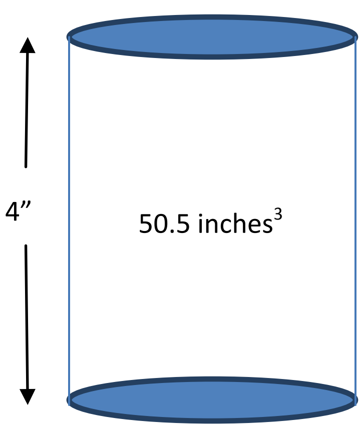
$(\text{height})^2 = 16$

Taking square root of both sides, we get

Height = 4cms

Note: there are two roots to √16 which are +4 and -4, but as this is a measure of length we know the answer must be the positive root.

5. You want to manufacture a cylindrical drink can of height 4 inches and a volume of 50.5 cubic-inches. What should its diameter be?



Volume of a cylinder = $\pi * \text{radius}^2 * \text{height}$

We are given the volume and the height of the cylinder. Therefore:

$50.5 \text{ inches}^3 = \pi * \text{radius}^2 * 4 \text{ inches}$

Dividing both sides by π * 4 inches, we get

$(50.5 \text{ inches}^3) / (4\text{inches} * \pi) = \text{radius}^2$ which can be simplified as

$4.02 \text{ inches}^2 = \text{radius}^2$

Taking the square root of both sides, we get:

Radius = 2.005 inches

Therefore diameter = 4.01 inches

Module 3

Exponents

Simplify the following:

1. $\frac{3a^3 - 2a^3 + (b - b)}{3b^2 - b^2 - b^2} = ?$

$$= \frac{a^3 + 1}{3b^2 - 2b^2} = \frac{a^3 + 1}{b^2}$$

2. $\frac{(\alpha^3 \beta^3)^{-1}}{(\alpha^{-3} \beta^{-3})} = ?$

$$= \frac{\alpha^{-3} \beta^{-3}}{\alpha^{-3} \beta^{-3}} = 1$$

3. $\frac{(f^3 g)^2}{(f g^{-3})^{-2}} = ?$

$$= \frac{f^6 g^2}{f^{-2} g^6} = \frac{f^6 * g^2}{f^{-2} * g^6} = f^{(6+2)} * g^2 * g^{-6} = f^8 * g^{-4} = \frac{f^6}{g^4}$$

4. $\frac{(p^{-4})^{-3}}{(p^{-3} q^5)^{-5}} = ?$

$$= \frac{p^{(-4*-3)}}{p^{(-3*-5)} * q^{(5*-5)}} = \frac{p^{+12}}{p^{+15} * q^{-25}} = p^{(12-15)} * q^{+25} = p^{-3} * q^{25} = \frac{q^{25}}{p^3}$$

5. $\frac{(\mu^{-42})^{(7-7)}}{(\beta^{-3} \Omega^5)^{(1.34-1.34)}} = ?$

$$= \frac{(\mu^{-42})^{(0)}}{(\beta^{-3} \Omega^5)^{(0)}} = \frac{1}{1} = 1$$

Roots and Radicals

Simplify the following expressions to the format **x * √y**. Example: √50 = √(2*25) = 5√2

1. √72 = ?

= √(2 * 2 * 2 * 3 * 3) = √(8 * 9) = 3√8

2. √49 = ?

= √(7 * 7) = 7√1

3. √48 = ?

= √(2 * 2 * 2 * 2 * 3) = √(4 * 4 * 3) = 4√3

4. √24 - √6 = ?

= √(2 * 2 * 2 * 3) - √6 = √(2 * 2 * 6) - √6 = 2√6 - √6 = √6

5. √63 + √28 + √112 = ?

= √(3*3*7) + √(2*2*7) + √(2*2*2*2*7) = 3√7 + 2√7 + 4√7 = 9√7

Solve for x:

1. √x = 4

Squaring both sides we get x = 16

2. $x^2 = 16$

Taking the square root of both sides we get x = ± 4

3. $(1 + x)^2 = 25$

Taking the square root of both sides we get

1 + x = 5 OR 1 + x = -5 therefore by subtracting 1 from both sides of both equations we get

x = 5 - 1 OR x = -5 – 1 thus

x = 4 OR x = -6

4. $1 + 2x + x^2 = 16$

A simple way to solve is using $(1 + x)^2 = 1 + 2x + x^2$ (you can confirm this by solving (1 + x) * (1 + x)) So we can substitute $(1 + x)^2$ for the LHS and get

$(1 + x)^2 = 16$

Taking the square root of both sides we have

$1 + x = 4$ OR $1 + x = -4$

Subtracting 1 from both sides of the two equations we get

$x = 3$ OR $x = -5$

5. $3 + 6x + 3x^2 = 48$

In this case you can factor out 3 from the LHS and get an expression similar to the last problem:

$3 * (1 + 2x + x^2) = 48$

Dividing both sides by 3 we get

$1 + 2x + x^2 = 16$ this is the same as the last problem and now you can solve to get

$x = 3$ OR $x = -5$

Polynomials

<u>Expand and simplify the following expressions:</u>

1. $(-7x^2 + 6x^4) – (3x^4 – 5x^7) = ?$

$= -7x^2 + 6x^4 – 3x^4 + 5x^7$

$= -7x^2 + 3x^4 + 5x^7$

2. (a + b) * (a + b) = ?

= a*a + a*b + b*a + b*b

$= a^2 + 2ab + b^2$

3. $(x + y)^2 * (x + y) = ?$

$= (x^2 + 2xy + y^2) * (x + y)$

$= x*x^2 + x*2xy + x*y^2 + y*x^2 + y*2xy + y*y^2$

$= x^3 + 2x^2y + xy^2 + x^2y + 2xy^2 + y^3$

$= x^3 + 3x^2y + 3xy^2 + y^3$

4. $(x + ay) * (x + by) = ?$

$= x*x + x*ay + by*x + ay*by$

$= x^2 + axy + bxy + aby^2$

5. Simplify by factoring: $x^2 – 3xy - 4y^2 = ?$

We want 2 numbers that add up to -3 and multiply to -4. In this case we will use -1 and 4 and re-write the above expression;

$= x^2 + xy – 4xy - 4y^2$ and factoring x out, and -4y out, we get;

$= x * (x + y) - 4y (x + y)$

$= (x – 4y) (x + y)$

<u>6. Use completing the square method to find the value of x. You are given that a, b, and c are all constants and x is the only variable.</u>

$ax^2 + bx + c = 0$ therefore dividing both sides by a we get

$$x^2 + \frac{bx}{a} + \frac{c}{a} = 0$$

then subtracting c/a from both sides, and adding $b^2/4a^2$ to both sides;

$$x^2 + \frac{bx}{a} + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$$

the LHS can now be expressed as a square term, and the RHS can be simplified:

$(x + b/2a)^2 = (-c/a) + (b^2/4a^2) = (-4a^2c + ab^2)/ 4a^3 = (-4ac + b^2)/4a^2$

Taking the square root of both sides we get

$x + b/2a = \sqrt{(b^2-4ac)} / 2a$ and subtracting b/2a from both sides we get:

$$x = \frac{-b \pm \sqrt{(b^2-4ac)}}{2a}$$

Module 4

Solving Linear Equations with Equalities

Solve for α in the following equations and wherever applicable round to one decimal:

1. $7\alpha - 12 = 3\alpha + 3$

$7\alpha - 3\alpha = 12 + 3$

$4\alpha = 15$

$\alpha = 15/4$

2. $2\alpha + 4 = \alpha + 5$

$2\alpha - \alpha = -4 + 5$

$\alpha = 1$

3. $11\alpha - 10 = 10\alpha + 3$

$11\alpha - 10\alpha = 10 + 3$

$\alpha = 13$

4. $2\alpha - 4 = 8 - \alpha$

$2\alpha + \alpha = 4 + 8$

$3\alpha = 12$

$\alpha = 4$

5. $½\, \alpha - 1 = \alpha/3 + 5$

$½\, \alpha - \alpha/3 = 1 + 5$

$(3\alpha - 2\alpha)/6 = 6$

$\alpha = 36$

With Inequalities

Solve for β in the following equations:

(Only multiplying or dividing by a negative number changes sign of the inequality)

1. $7\beta < 14$

   Dividing both sides by 7, we get $\beta < 2$

2. $13\beta \leq 11$

   Dividing both sides by 13, we get $\beta \leq 11/13$

3. $-6\beta + 12 < -14 + 3\beta$

   Subtracting 3β and 12 from both sides:

   $-6\beta - 3\beta < -14 - 12$

   $-9\beta < -26$

   Dividing -9 from both sides, we flip the sign:

   $\beta > 26/9$

4. $3\beta + 1 \geq -2 - \beta$

   Adding β to both sides and subtracting 1 from both sides:

   $3\beta + \beta \geq -2 - 1$ OR $4\beta \geq -3$

   Dividing by 4 on both sides:

   $\beta \geq -3/4$

5. $4\beta + 4 \geq 16 - \beta^2$

   Adding β^2 to both sides of the inequality we get;

   $\beta^2 + 4\beta + 4 \geq 16$

   As the LHS represents a squared term, we can simplify

   $(\beta + 2)^2 \geq 4^2$ so taking the square root we know $\beta + 2 \geq 4$ or $\beta + 2 \leq 4$

   If $\beta + 2 \geq +4$; then $\beta \geq 2$ and when $\beta + 2 \leq -4$; then $\beta \leq -6$

Introduction to Statistics

1. Find the mean and the median of the following numbers: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

Mean: $\mu = \frac{\sum x}{N} = \frac{(1+2+3+4+5+6+7+8+9+10)}{10} = 5.5$

Median: Middle pair of number is 5 and 6, median is 5.5

2. What is the median of the following numbers? 4, 5, 6, 13, 11, 2, 9

Median: arrange in ascending order and pick middle number: 6

3. What is mode of the following numbers? 4, 13, 4, 5, 2, 13, 1, 6, 8, 9, 13, 13, 12, 5, 9, 6

Mode: most frequent number which is 13

4. What is the mean, median and mode of the following numbers? 5,4,3,4,1,2,2

Mean is 3

Median is 3

Mode is 2 and 4, the distribution is bi-modal

5. In a class there are 8 students who scored 85, 64, 98, 83, 78, 95, 77, 100 points on their exam. What is the mean score of the class? What is the median score?

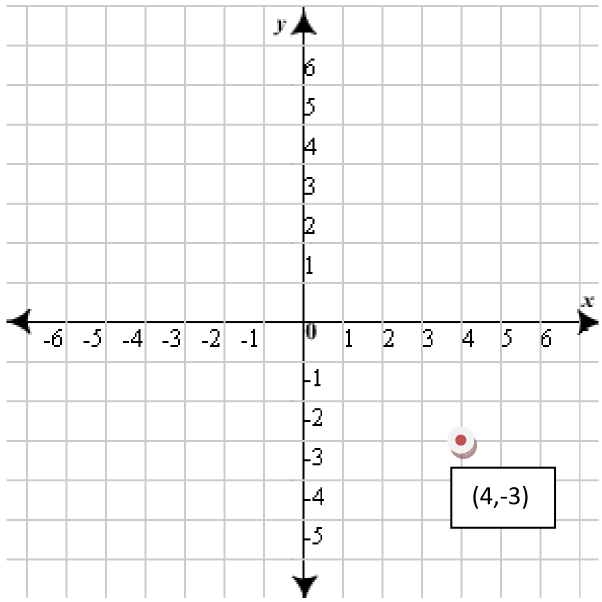
Mean score of the class is 85 points

Median score of the class is 84 points.

Module 5

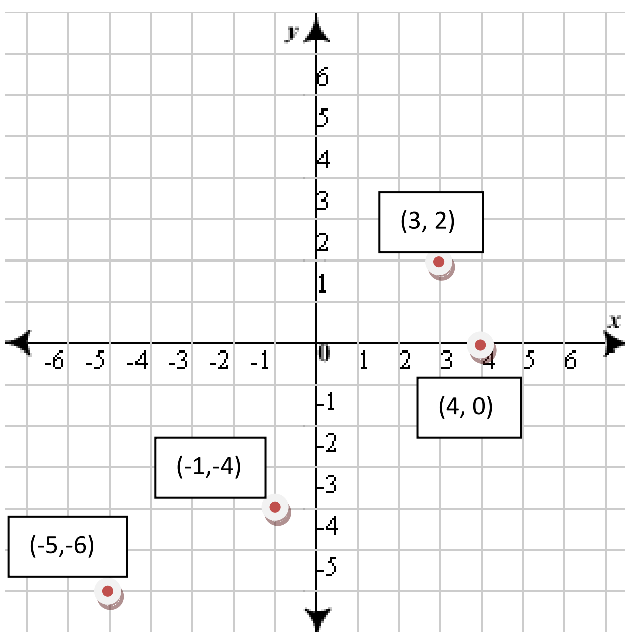
Graphs of Linear Equations

1. In which quadrant is the point (4, -3)

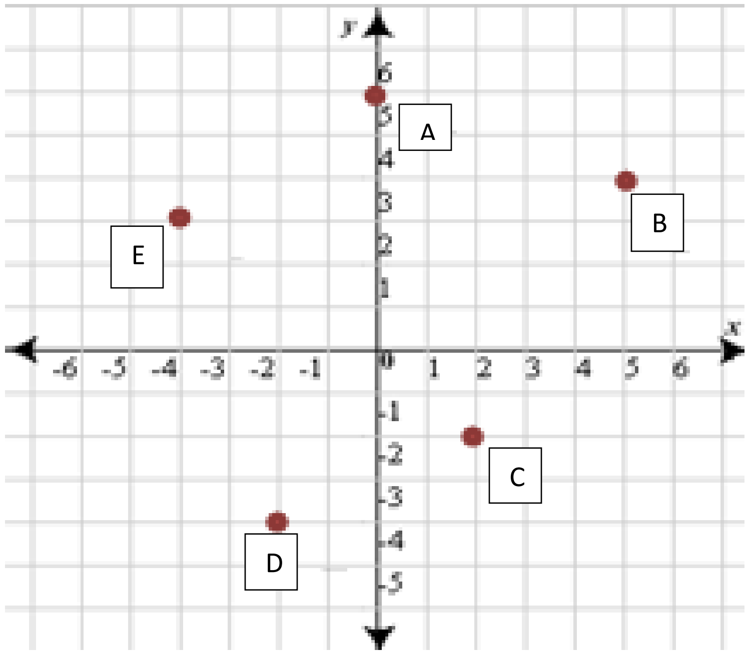


Point (4,-3) is in Quadrant 4.

2. Graph the points (-1, -4); (3, 2); (4, 0); (-5; -6)

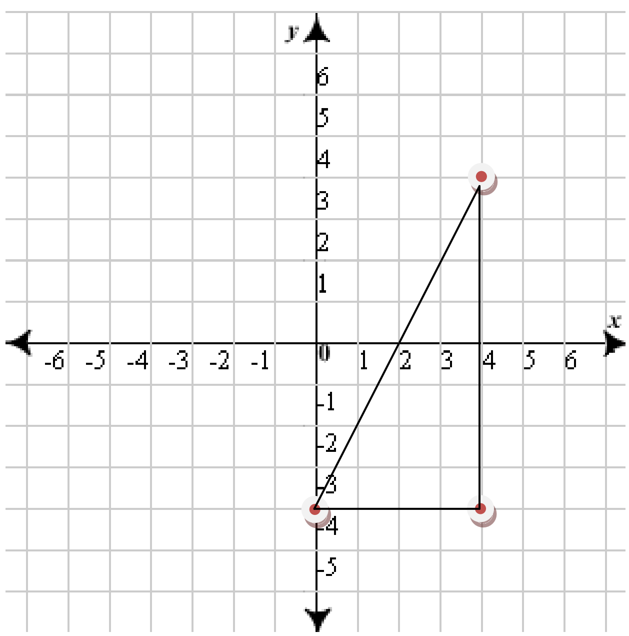


3. Identify the following points from the graph:



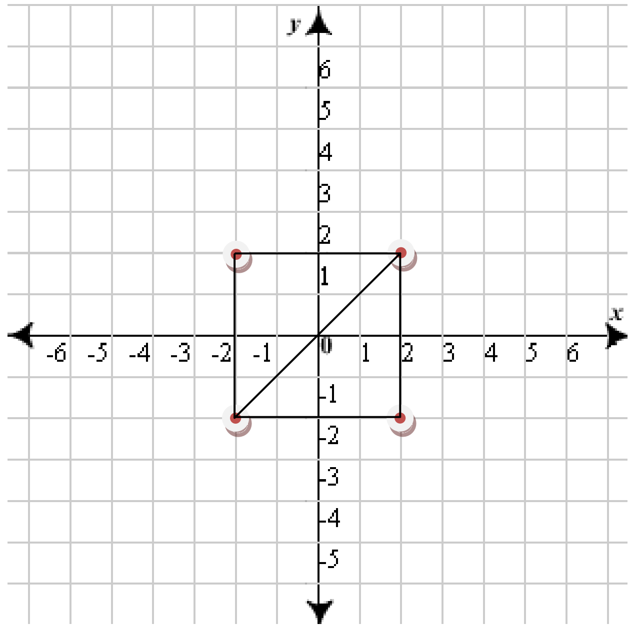
A is (0, 6); B is (5, 4); C is (2, -2); D is (-2, -4); E is (-4, 3)

4. What is the area of the triangle whose vertices are marked out by the ordered pairs (4, 4); (4, -4); and (0, -4).



Area of triangle = 0.5*base*height = 0.5*(4-0)*(4-(-4)) = 0.5*4*8 = **16unit^2**

5. What is the length of the diagonal of a square that has its vertices as the ordered pairs (-2, -2) ; (2, 2); (-2, 2); (2, -2)?



Using Pythagorean theorem we know that $side^2 + side^2 = hypotenuse^2$, therefore

$Hypotenuse^2 = 4^2 + 4^2 = 32$ therefore hypotenuse is $\sqrt{32} = 5.66$ units

Graphing Lines

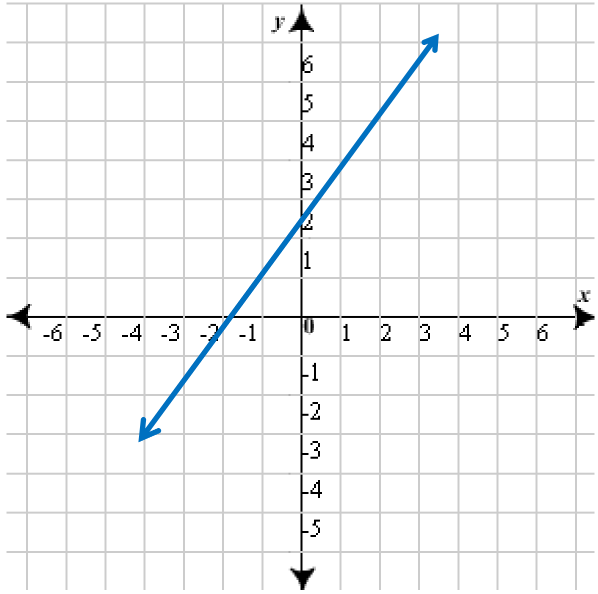
1. Which of the following ordered pairs is a solution to the equation y = x + 4?

(-2,2) (-1,1) (3, 5) (4, 2) (5, 9)

Substitute each ordered pair into the equation to find the solution. There are two solutions: both (-2, 2) and (5, 9) are solutions to y = x + 4

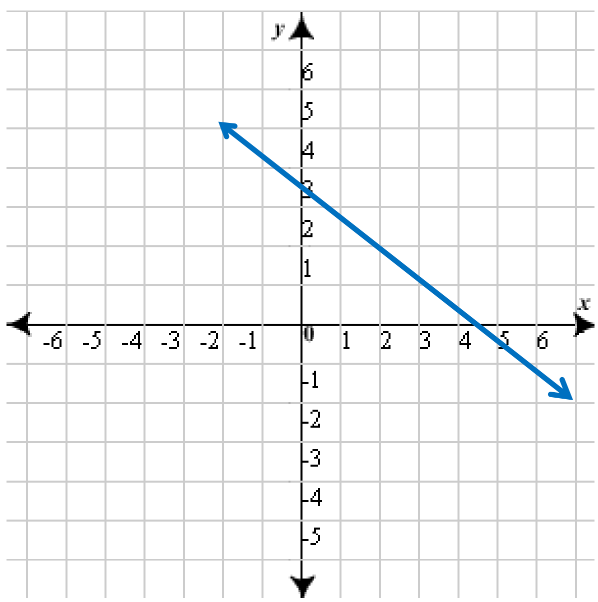
2. Which of the following ordered pairs is a solution to the line graphed below?

(-2,2) (-1,1) (3, 5) (4, 2) (5, 3)



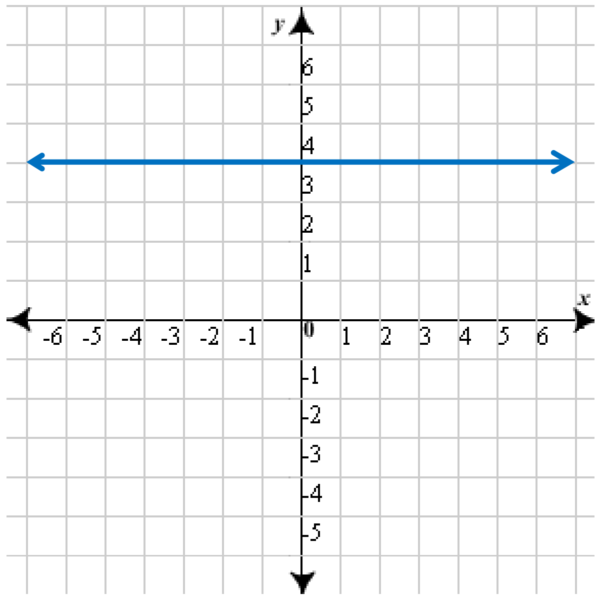
To be a solution, the ordered pair must be on the line. Therefore of the ordered pairs given only (-1, 1) is a solution. Alternately, you can also calculate the slope of the line, and its y-intercept to derive an equation of the line in point-slope form then substitute the values of the ordered pairs in to see which ordered pair is a solution to the line.

3. Based on the line graphed below, what is the value of y when x = 2?



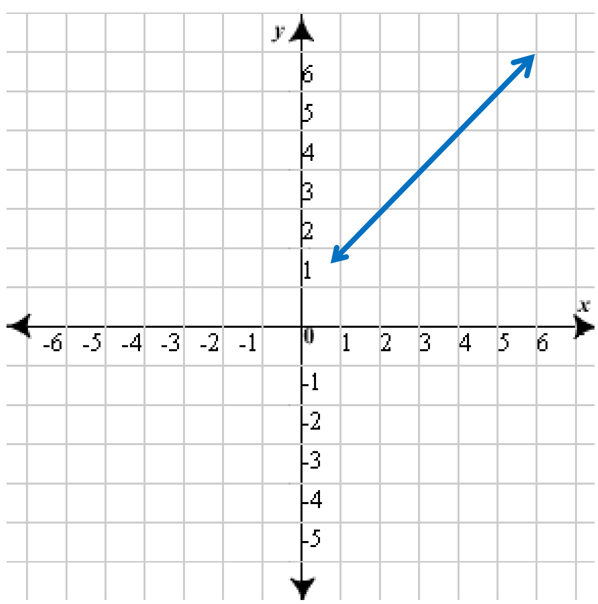
The line passes through point (2, 2) so when x = 2, we know y=2

4. Based on the line graphed below, what is the value of x when y = 4?



The line is horizontal to the x-axis at y = 4, therefore there are infinite values that x can take which hold true for y=4.

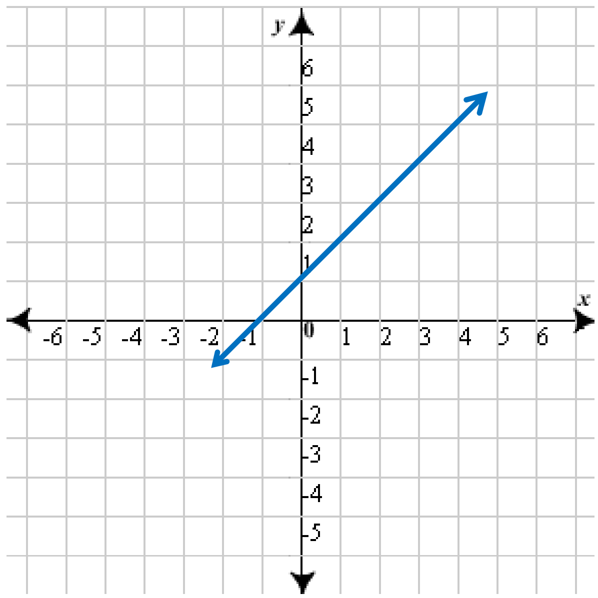
5. The following graph shows the relationship between price of bread (on y-axis) and consumption of bread (on x axis). What happens to consumption of bread as its price increases? Is it increasing, decreasing, or staying the same? Is the result what you would expect?



According to the graph, consumption of bread increases as price increases. This is not the result we would expect as typically we would expect people to consume less of goods as the price of the good increases (demand curves are downward sloping).

Slopes and Intercepts

1. What is the slope of the line below? What is its y-intercept? What is its x-intercept?

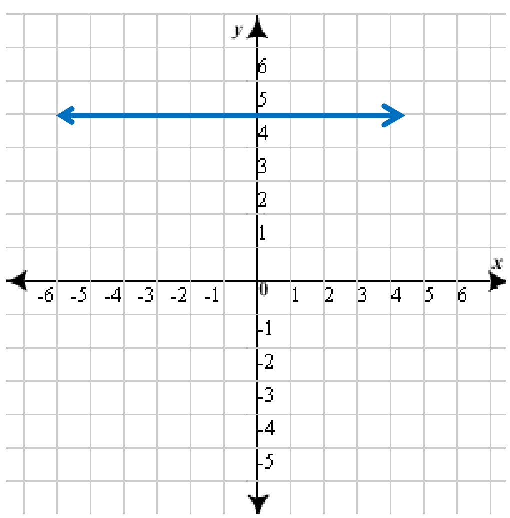


Slope: 1

y-intercept: (0, 1)

x-intercept : (-1, 0)

2. What is the slope of the line below? What is its y-intercept? What is its x-intercept?

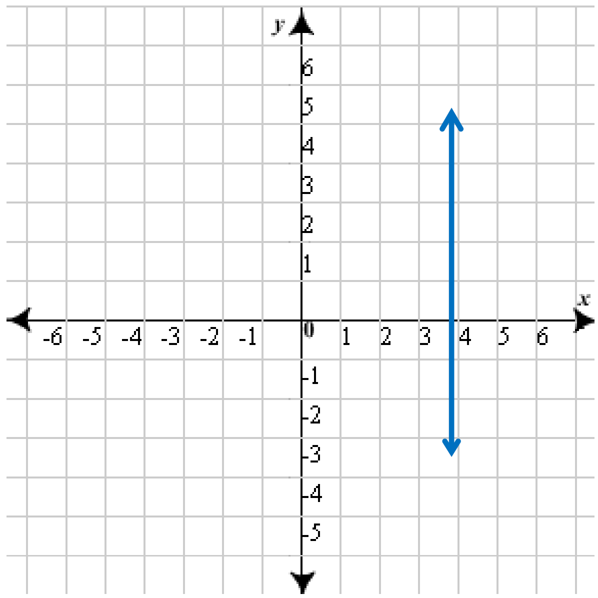


Slope: 0

y-intercept: (0, 5)

x-intercept: Does not exist as line is parallel to x-axis.

3. What is the slope of the line below? What is its y-intercept? What is its x-intercept?

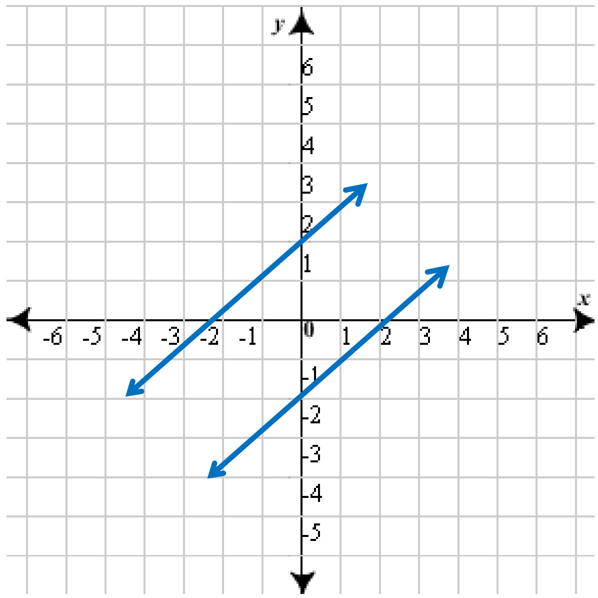


Slope: undefined

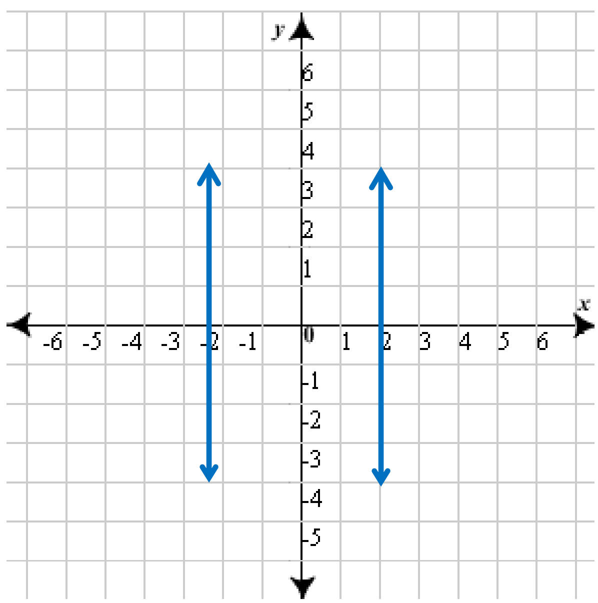
y-intercept: does not exist as line is parallel to y-axis

x-intercept: (4, 0)

4. Graph a line parallel to the given line and with a y-intercept at (0,2)?

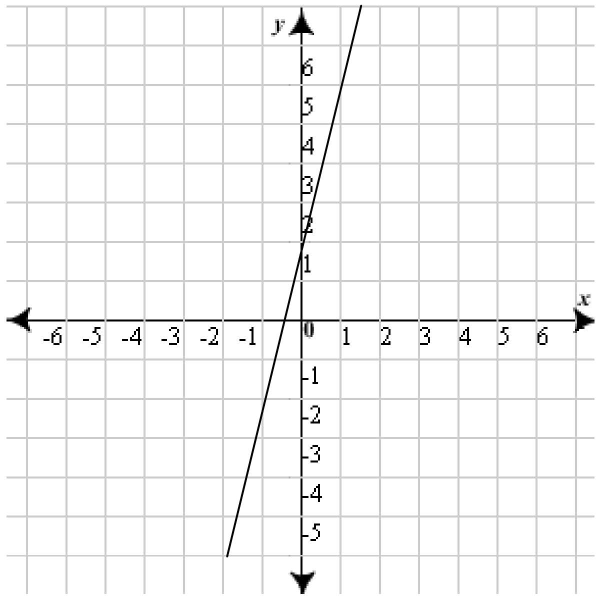


5. Graph a line parallel to the given line and with a x-intercept at (2,0)?

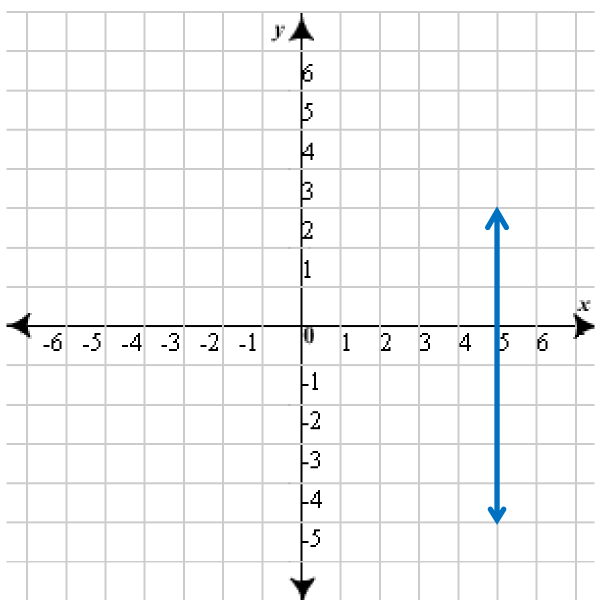


Equations of a Line

1. Graph a line whose equation is 3y = 12x + 6

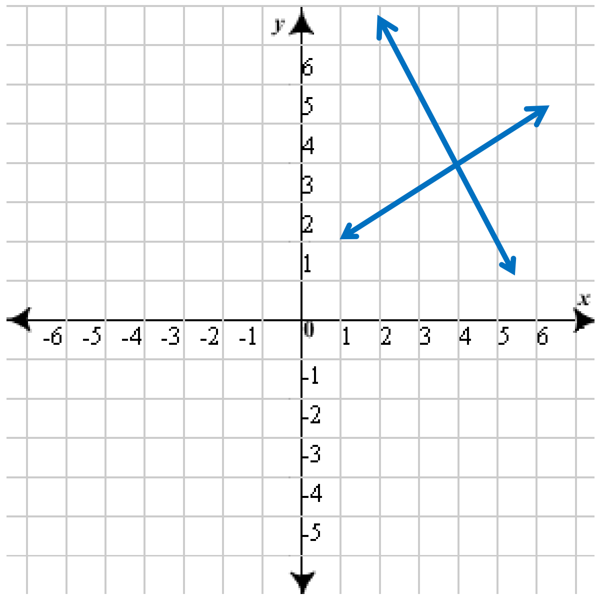


2. What is the equation of the line graphed below?



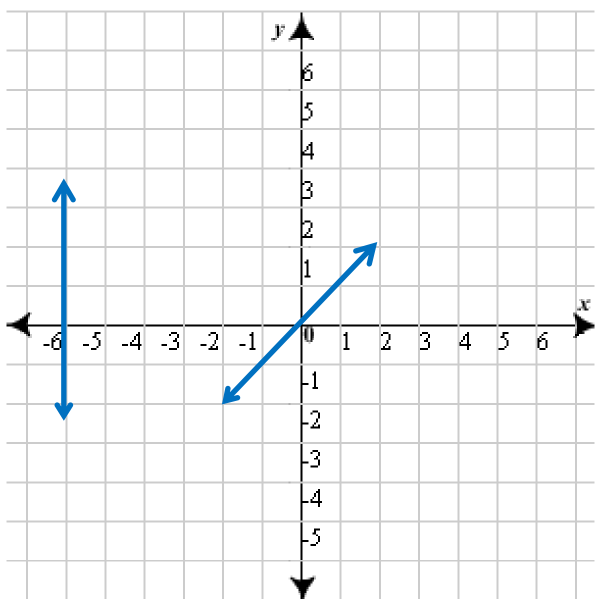
x = 5

3. Name a point that satisfies the equations of both the lines below.



(4, 4)

4. Name a point that satisfies the equations of both the lines below.

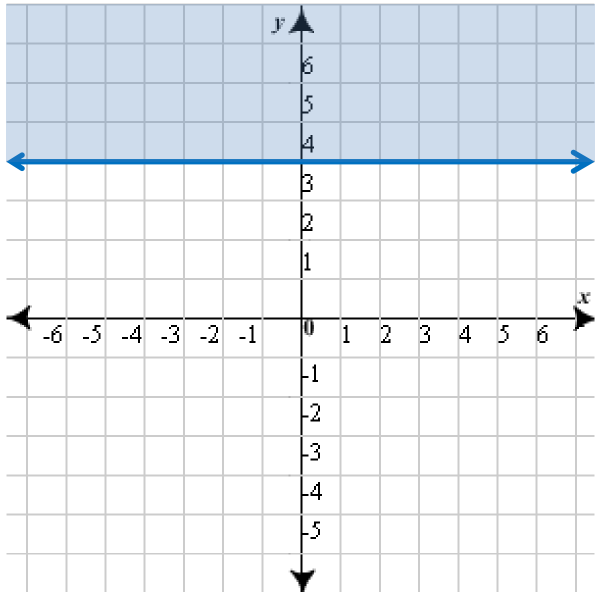


Equation of line 1 is x = -6

Equation of line 2 is y = x

Point that satisfies both equations is (-6, -6)

5. What is the inequality represented in the graph below?



The equation of the line above is y = 4 and the shaded region represent all points above that line including the line itself. So the inequality represents $y \geq 4$

1. Name 4 points that lie on the perimeter of a circle whose equation is $\mathbf{x^2 + y^2 = 16}$

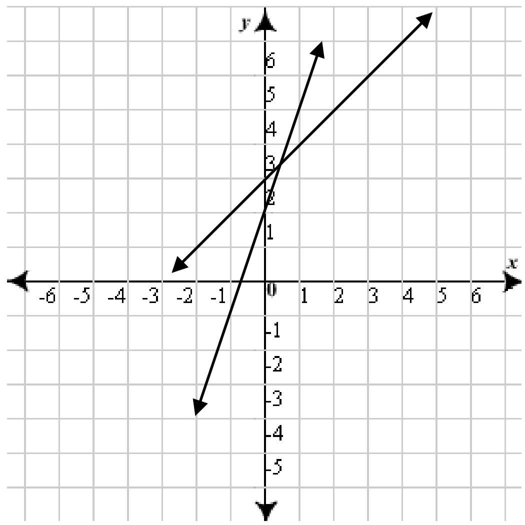
Four points that satisfy the equation are (0, 4); (4, 0); (0, -4); and (-4, 0)

Module 6

Systems of Equations

1. Graph this system of equations and solve

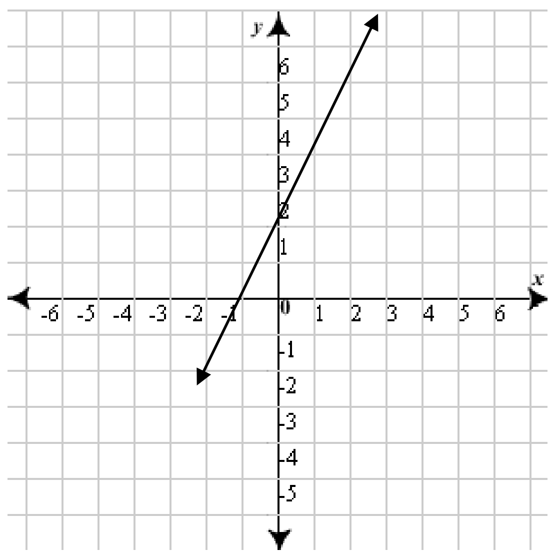
(i) y = 3x + 2 (ii) y = x + 3



Answer: (0.5, 3.5)

2. Graph this system of equations and solve

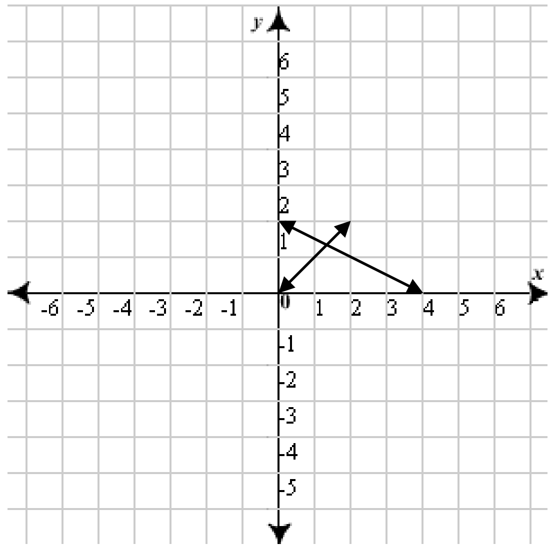
(i) 2y = 4x + 2 (ii) y = 2x + 1



Answer: The two lines share all the same points and hence the two are not independent and share an infinite number of solutions.

3. Graph the lines represented by the following ordered pair and solve the system of equations

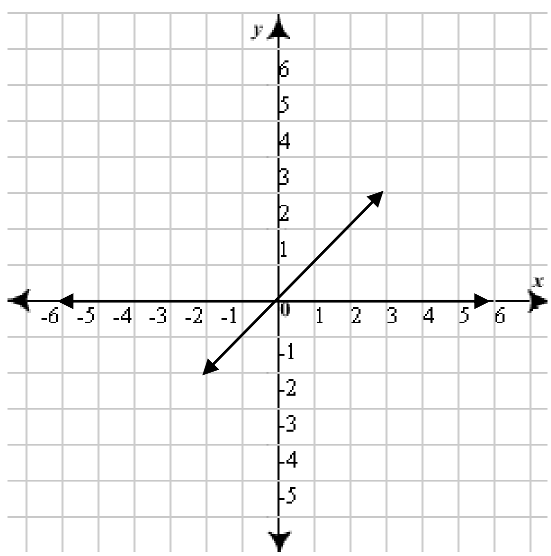
(i) (0,2); (4, 0) and (ii) (1,1); (2,2)



Equation of line (i) is y = -½ x +2 and equation of line (ii) is y=x therefore the common solution is the ordered pair (4/3 , 4/3)

4. Graph the lines represented by the following ordered pair and solve they system of equations

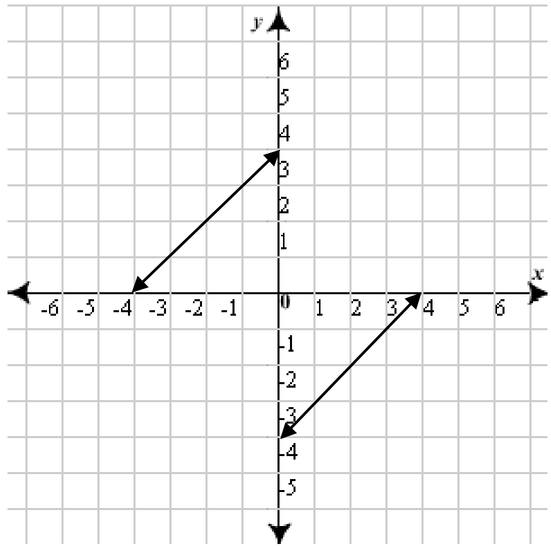
(i) (0,0); (4, 0) and (ii) (-2,-2); (3,3)



Equation of line (i) is y = 0 and equation of line (ii) is y = x so the solution to the system is the ordered pair (0, 0)

5. Graph the lines represented by the following ordered pairs. Is there a solution for this system of equations?

(i) (-4, 0); (0, 4)   (ii) (0, -4); (4, 0)



The two lines are parallel and hence there is no solution to this system.

Solving Systems of Equations using Graphical Intuition

1. How many solutions exist for this system of equations? (One solution; two solutions; infinite solutions; or, none)

(i) 15x – 3y = -27   (ii) y = -3 – 3x

Substituting (ii) in (i) we get

15x -3(-3 -3x) = -27 and simplifying this further we get x =3/2. Substituting x=3/2 back into equation (ii) we get y=-15/2 = -7.5

Therefore there is only one solution to this system of equations which is (3/2, -15/2)

2. How many solutions exist for this system of equations? (One solution; two solutions; infinite solutions; or, none)

(i) 2y = -3x + 8   (ii) 8y = -12x + 32

Y dividing both sides of equation (ii) with 4, we get 2y = -3x + 8 which is the same as equation (i). Therefore the two equations are not linearly independent and share an infinite number of solutions.

3. How many solutions exist for this system of equations? (One solution; two solutions; infinite solutions; or, none)

(i) $2y = -3x + 8$ (ii) $8y = -12x + 80$

Dividing equation (i) by and equation (ii) by 8 we see that both lines have the same slope, but different y-intercepts. This indicates that the two lines are parallel to each other and share no point in common, i.e. this system of equations has no solutions.

4. Fill in the blanks below to complete the equation of a line that shares infinitely many solutions with a line of equation

$3x – 2y = 12$

Ans: __3/2__ x - __1__ y = 6

5. Write the equation to a line that is parallel to the line $y = 5x + 3$ and passes through the point (0, 4)

$y = 5x + 4$

Solving Systems of Equations using Substitution

1. (i) $2x + 5y = -4$ (ii) $y = -4x + 10$

Substitute (ii) in (i) and simplify to get

Answer: $x = 3$ and $y = -2$

2. (i) $-3x + 2y = 12$ (ii) $x = -3y + 7$

Substitute (ii) in (i) and simplify to get

Answer: $x = -2$ and $y = 3$

3. (i) $2x – 4y = -6$ (ii) $x = 5y - 6$

First divide equation (i) by 2 on both sides then substitute (ii) in (i) and simplify to get

Answer: $x = -1$ and $y = 1$

4. (i) -5x – 3y = 7 (ii) y = -x +1

Substitute (ii) in (i) and simplify to get

Answer: x = -5 and y = 6

5. (i) 6x – 6y = 6 (ii) y = 5x - 9

First divide equation (i) on both sides by 6 and then substitute (ii) in (i) and simplify to get

Answer: x = 2 and y = 1

Solving Systems of Equations using Elimination

1. (i) 3x – 3y = 6 (ii) -3x - 5y = -46

Adding (i) and (ii) we get -8y = -40 so we know y =5 and then substituting value of y back into equation (i) we get x = 7

Answer: x = 7 and y = 5

2. (i) 6x + y = 27 (ii) -5x – y = -23

Adding (i) and (ii) and simplifying we get

Answer: x = 4 and y = 3

3. (i) -2x + y = -5 (ii) -3x – y = -25

Adding (i) and (ii) and simplifying we get

Answer: x = 6 and y = 7

4. (i) -2x - 3y = -44 (ii) 2x + 5y = 64

Adding (i) and (ii) and simplifying we get

Answer: x = 7 and y = 10

5. (i) -2x - 3y = -15 (ii) 2x + 5y = 17

Adding (i) and (ii) and simplifying we get

Answer: x = 6 and y = 1

Systems with three variables

1. (i) $3x - y + z = 2$ (ii) $2x + 2y - z = 12$ (iii) $x + y - z = 6$

Add (1) and (3)

$4x = 8$

$\underline{x = 2}$ ....(4)

Substitute (4) in (2)

$2y - z = 8$ .... (5)

Substitute (4) in (3)

$y - z = 4$..............................(6)

Subtract (6) from (5)

$\underline{y = 4}$ ....(7)

Substitute (4) and (7) into (3)

$\underline{z = 0}$

Answer: $x = 2$; $y = 4$; and $z = 0$

2. (i) $x - 2y + 3z = 7$ (ii) $2x + y + z = 4$ (iii) $-3x + 2y - 2z = -10$

Answer: $x = 2$; $y = -1$; and $z = 1$

3. (i) $2x - 4y + 5z = -33$ (ii) $4x - y = -5$ (iii) $-2x + 2y - 3z = 19$

Answer: $x = -1/2$; $y = 3$; and $z = -4$

4. (i) $x + y + z = 2500$ (ii) $y - z = 6000$ (iii)$.6x + 0.7y + 0.8z = 162$

Answer: $x = 9420$; $y = -460$; and $z = -6460$

5. (i) $4x + y - 2z = 0$ (ii) $2x - 3y + 3z = 9$ (iii) $-6x - 2y + z = 0$

Answer: $x = 3/4$; $y = -2$; and $z = 1/2$

Module 7

Functions

1. Write the equation of function that represents the relationship between cost of tuition and number of course credits taken, if each course credit costs $345. What would be the cost of tuition if you registered for 5 course credits?

Let x be the number of course credits. Then;

*f(x) = $345*x   ; When x = 5, f(5) = 345*5 = **$1725***

2. If you toss a ball of the top of founders hall building, its height (in feet) at any time (in seconds) is represented by the function $h(t) = 250 – 8t^2$. What is the height of the ball two seconds after you drop it?

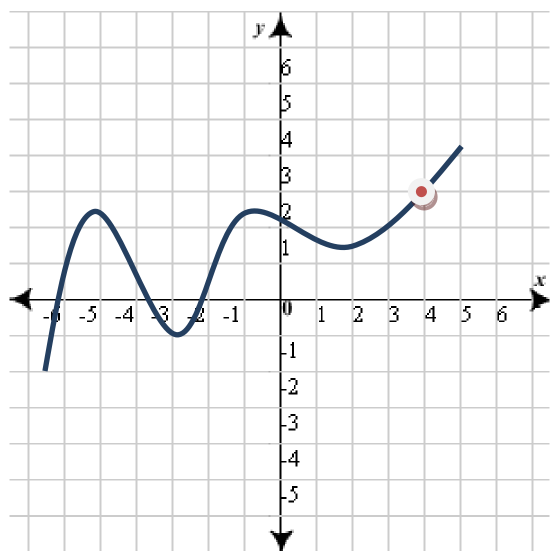
$h(2) = 250 – 8(2^2) = 250 – 32 =$ **218 feet.**

3. Your bank is offering you a certificate of deposit (CD) option at 6% for 4 years. If you invest $5,000 today, how much will you have in the CD when it matures? Formula for compound interest is:

Future Value = Present Value * $(1 + \text{interest rate})^{\text{time in years}}$, round your answer to the hundredth place.

Future Value = $5000 * $(1 + 0.06)^4$ = **$21,200**

4. In the graph below what is f(4)?



Answer: f(4) = 3

5. For f(h) = | -3h + 10 | what is f(5)?

f(5) = |-3*5 + 10 | = | -15 + 10 | = |-5| = **<u>5</u>**

Recognizing Functions

<u>Answer if the following relations represent a function or not.</u>

1. The value of y is always 13 less than x.

Yes. y = x – 13 this is a function as it fits the definition of a function and there is only one y value for any one x.

2. y = 3x + 4

Yes.

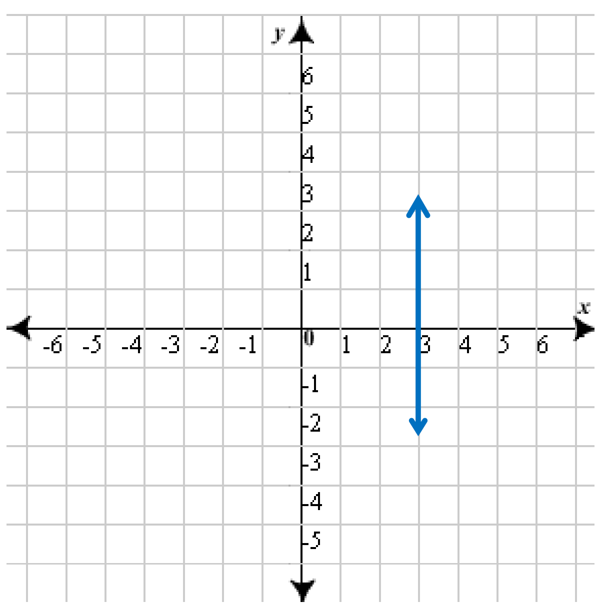
3. $x^2 + y^2 = 1$

No, as we can have more than one value of y for a given x.

4. $y^2 = x$

No, as we can have more than one value of y for a given x. For instance, x = 4, y can be +2 or -2.

5. Is y a function of x in the following graph?



No, as we can have more than one value of y for a given x.

6. John has a very unique method of making his coffee. If he is making between 1 to 5 cups of coffee (outputs) he will always put 5 tablespoons of coffee ground (inputs) into his machine. However if he makes more than 5 cups, he adds one more table spoon of coffee per additional cup. Can the number cups of coffee he makes be represented as a function of the number of tablespoons of coffee grounds he used?

No cannot be represented as a function as, for different values of inputs (x variable) we get the same output (y variable) when making 5 cups of coffee or less.

7. At your local grocer you can buy a candy bar for $1; milk for $2; cookies for $0.50; and pencils for $0.25. Can you represent your price as a function of the products bought?

Yes, as it follows the definition of a function.

8. Given the table below, can you represent y as a function of x?

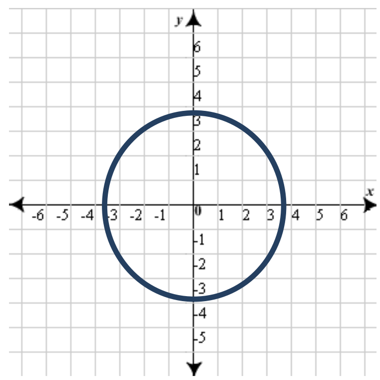
y	x
0	15
5	12
10	4
15	15
20	12

No, because for the same input (15 or 12), we would get multiple outputs, thus violating the definition of a function.

Brief Intro to Conic Sections

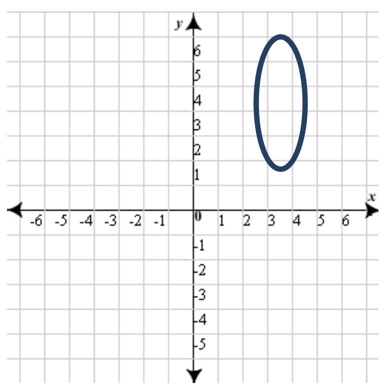
<u>Identify the following conic sections: Is it circle? Hyperbola? Parabola? Or an Ellipse?</u>

1.



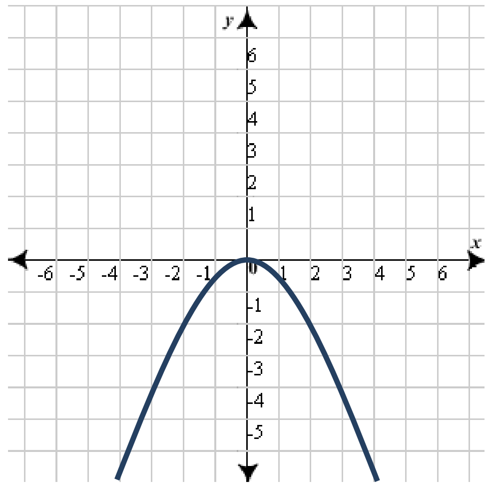
Circle

2.



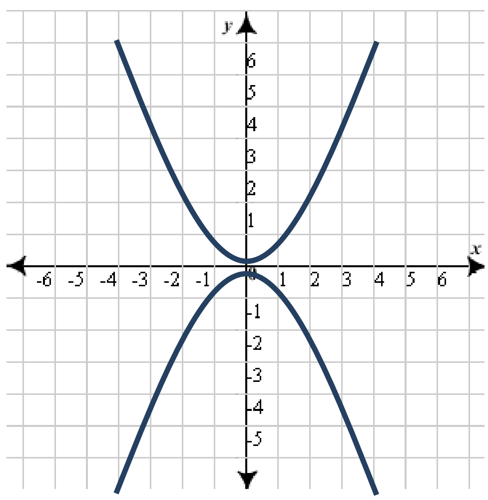
Ellipse

3.



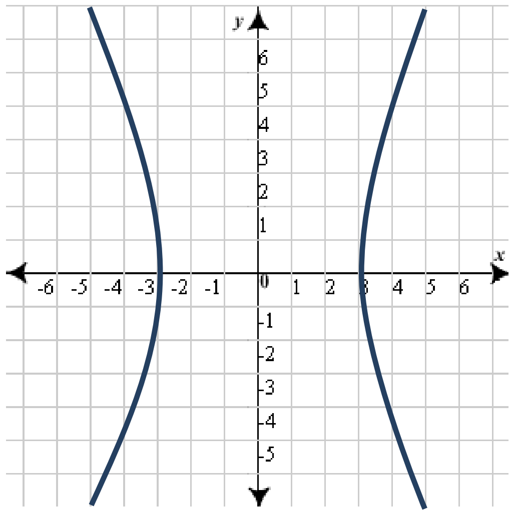
Parabola

4.



Hyperbola

5.



Hyperbola

Module 8

Derivatives using Power Rule

Take the first derivative of the following

1. $f(x) = 12x^8$

$f'(x) = 12*8*x^{8-1} = \mathbf{96x^7}$

2. $f(x) = -13x^{-5}$

$f'(x) = -13 * -5 * x^{-5-1} = 65x^{-6}$

3. $f(x) = 1/x^3 = x^{-3}$

$f'(x) = -3x^{-4}$

4. $f(x) = 12/x^4 = 12x^{-4}$

$f'(x) = 48x^{-5}$

5. $f(x) = 11/7$

$f'(x) = 0$

Derivatives using Product Rule

$$D\{f(x)g(x)\} = f(x)g'(x) + f'(x)g(x)$$

Take the first derivative of the following

1. $y = (x^3 + 2x + 1)*(2x + 2)$

$dy/dx = (3x^2+2)*(2x+2) + (x^3+2x+1)*2$

$dy/dx = 8x^3 + 6x^2 + 8x + 6$

2. $y = x^4 * (4x - 2)$

$dy/dx = 20x^4 – 8x^3$

3. $y = (x^2 – 4x) * (1 + 3x^2)$

$dy/dx = 12x^3 – 36x^2 + 2x - 4$

4. $y = (x – 2) * (x^3 – 2)$

$dy/dx = 4x^3 – 6x^2 -2$

5. $y = x^3 * (2x + 5)$

$dy/dx = 8x^3 + 15x^2$

Derivatives using Chain Rule

$$D\{f(g(x))\} = f'(g(x))\ g'(x)$$

1. $y = (2x + 3)^2$

$dy/dx = 8x + 12$

2. $y = \sqrt{(x^2 + 3x + 2)}$

$$\frac{dy}{dx} = \frac{2x + 3}{2\sqrt{(x^2+3x+2)}}$$

3. $y = (21 + 4x^2)^5$

$dy/dx = 40x(4x^2 + 21)^4$

4. $y = \frac{3x^2 - 4x}{x^3}$

$dy/dx = (8 - 3x) / x^3$

5. $y = \frac{5x - 2x^2}{x^{-3}}$

$dy/dx = 20x^3 - 10x^4$

Partial Derivatives

1. Take the first order partial derivative of f(x,y) with respect to x for

$f(x,y) = x^2 + 2xy + y^2$

$\partial f/\partial x = 2x + 2y$

2. Take the first order partial derivative of f(x,y) with respect to y for

$f(x,y) = (xy^3 + 2x + 1)*(2y + 2)$

$f_y(x,y) = 8xy^3 + 6xy^2 + 4x + 2$

3. Take the **second order** partial derivative of f(x,y) with respect to x for

$f(x,y) = x^3 – y^3$

$f_x(x,y) = 3x^2$

$f_{xx}(x,y) = 6x$

4. Take the **second order** partial derivative of f(x, y) with respect to y for

$f(x,y) = x^2y + 2x + y$

$f_y(x,y) = x^2 + 1$

$f_{yy}(x,y) = 2x$

5. Take the **first order** partial derivative with of f(x,y) with respect to x and the **second order** partial derivative of f(x,y) with respect to y for

$f(x,y) = x^3y^3 + 3x^3y$

$f_x(x,y) = 3x^2y^3 + 9x^2y$

$f_{xy}(x,y) = 9x^2y^2 + 9x^2$

Optimization

1. Maximize utility for the given utility function $U(q) = 1000q – q^2$

$U'(q) = 1000 – 2q$

First Order Condition (F.O.C):

$1000 – 2q = 0$

Therefore **q = 500**

2. Find the critical point of the function $g(z) = z^2 + 4z + 3$

$g'(z) = 2z + 4$

First Order Condition (F.O.C)

$2z + 4 = 0$

Therefore z = -2 or the critical point is at **z=-2**

3. Find the critical point of the function $p(q) = q^3 – 3q^2 + q$

$U'(q) = 3q^2 - 6q + 1$

First Order Condition (F.O.C):

$3q^2 - 6q + 1 = 0$

Therefore **critical points are at $q = 1 - \sqrt{(2/3)}$ or $q = 1 + \sqrt{(2/3)}$**

4. Find the critical point with <u>respect to x</u> for $U(x,y) = 20x + 80y - x^2 - 2y^2$

$U_x(x,y) = 20 - 2x$

F.O.C: $20 - 2x = 0$

Therefore $x = 10$ is a critical point

5. Find the critical point with <u>respect to y</u> for $U(x,y) = 20x + 80y - x^2 - 2y^2$

$U_y(x,y) = 80 - 4y$

F.O.C.: $80 - 4y = 0$

Therefore $y = 20$