Extending the Input-Output Table Based on Firm-level Data

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Abstract

This paper proposes a general method to extend a standard input-output (IO) table to incorporate firm heterogeneity when portraying the domestic segment of global value chains in a country. We develop a quadratic optimization model to estimate an extended IO table that reports inter-sector transactions between different types of firms in an economy, using information from standard IO tables along with various linear constraints implied by sector-level statistics and firm-level data. The proposed method permits the computation of standard errors of all values in the estimated IO tables, inferred from bootstrapped samples of the underlying firm-level data. As an illustration, we implement our model using Chinese IO tables and firm census data. We then use the estimated IO tables to compute the direct and indirect domestic value added in exports of different firm types in China. Based on our reconciled data sets for 2007 and 2010, we find that both state-owned enterprises (SOE) and small and medium enterprises (SME) in China have much higher value-added exports (VAX) to gross exports ratios, compared to the rest of the economy. While the VAX ratio of China’s aggregate exports increased by about 9\% between 2007 and 2010, SOE’s and SME’s VAX increased by 47\% and 27\%, respectively.

\textbf{Key words:} value-added trade; global value chains; quadratic optimization; intra-national trade

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1. Introduction

The expansion of global value chains (GVC) has made gross trade statistics increasingly inaccurate in describing the pattern of world trade. To tackle this discrepancy, a large and growing literature has proposed methods to use input-output (IO) tables to gauge the actual value added by different countries in global value chains (GVC) (e.g., Hummels, Ishii, and Yi, 2001, Johnson and Noguera, 2014 and Koopman, Wang, and Wei, 2014). The literature has so far given little attention to an equally important part of GVC – the domestic segment within a country. International trade affects not only the sectoral allocation of resources, but also the spatial distribution of domestic factors of production (Ramondo, Rodríguez-Clare, and Saborio, 2016, Redding, 2015). Adding to these realities is firm heterogeneity, which has been shown to play an important role in shaping the patterns and thus the welfare and distribution effects of trade (Melitz, 2003; Melitz and Redding, 2015). How demand or supply shocks to trade propagate across sectors and regions in a country? Do small and medium-sized firms benefit from GVC through indirect participation even though they do not export directly? Which region of a country benefits the most from trade? Unfortunately, standard IO tables, which typically report only inter-sector transaction flows, do not provide sufficient information to answer these important questions. Survey data on intra-national trade, such as the US Commodity Flow Survey, are usually limited in coverage or simply non-existent for most countries.

We propose a method to extend a standard input-output (IO) table into a detailed account using firm-level data. Specifically, we develop a quadratic optimization model to estimate an extended IO table that reports inter-sector transactions between different types of firms in a country, using standard national IO tables and linear constraints implied by sector-level statistics and firm-level data. The idea is to minimize a quadratic penalty function with arguments equal to the values of the extended IO table, subject to a series of accounting identities and adding up constraints according to economic theory and aggregate statistics (e.g., industry-level exports and imports). The method we propose is general enough to be augmented to study any measurable dimensions of firm characteristics, including their geographic locations. The method can also be applied to portraying the domestic input-output linkages of a country’s exports, including the distribution of value-added exports across different firm types, as long as IO tables, basic firm balance-sheet data and import and export statistics by firm type and sector are available. Importantly, the method permits the construction of standard errors of all values in the estimated IO tables, using samples bootstrapped from the underlying firm-level data.

As an illustration, we implement our program using China’s IO tables for 2007 and 2010 and
census data for both manufacturing and service firms from 2008. Based on ownership type and size, we categorize firms into four groups: state-owned enterprises (SOE), foreign invested enterprises (FIE), large private enterprises (LP), and small and medium private enterprises (SME). We then estimate the volume of transactions in the extended tables using our proposed constrained quadratic optimization; and use the extended tables to portray the pattern and evolution of the domestic segment of GVC inside China. In particular, we quantify the contributions of different domestic input-output channels through which Chinese value added exports (VAX) were generated.

We find that in China, SOE’s VAX are significantly larger than their gross exports, contrasting with the common view about China’s low VAX to gross exports ratio (Chen et al., 2012; Koopman, et al. 2012). Specifically, the VAX ratio of SOE is estimated to be 1.2 in 2007, which increased by 50% to 1.8 in 2010, compared to around 0.35 for FIE in both years. Among private firms, large firms’ VAX ratio is around 0.7 for both years, while SME’s VAX ratio exceeded 1 for both years, and increased from 1 to 1.3 between 2007 and 2010. In other words, for both SOE and SME, their actual (value added) exports in value-added terms have been larger than what the official gross export statistics suggest.

One may be concerned about the validity of these results, as after all, our estimation depends on the linear constraints and initial conditions that we impose in our optimization model. To this end, we use bootstrapped firm samples to construct a range of constraints and initial conditions, which are used for constructing thousands of simulated IO tables. Based on a large number of estimated IO tables, we then compute standard errors and confidence intervals of the estimates of both direct and indirect VAX by firm type. The estimates appear to be less precise if the underlying observations within each firm category used to construct the initial values in the penalty function or the constants in the linear constraints are more dispersed. Naturally, the precision of the estimates will be lower if more firm dimensions are added in the estimation model, as fewer observations are drawn within each cell from the firm census.

Another advantage of extending a conventional IO table into sub-accounts based on micro data is that we can analyze transactions between different firm types in the domestic segment of GVC. We find that in China, indirect exports (i.e., exporting through other firms) accounted for about 80% of SOE’s VAX in 2007, which further increased in 2010. Of these indirect exports, about 40% was

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5 Previous research has extended an IO table to take into account differences between processing and non-processing trade in China and Mexico (e.g., Johnson and Noguera, 2014 and Koopman, Wang, and Wei, 2014).

6 The same approach has been used to split a national IO table for China into regional IO tables (Koopman, Meng, and Wang, 2014). These regional IO tables can be used to assess the effects of trade liberalization on intra-national trade and regional income disparity. See Tombe and Zhu (2015) for such an analysis for China.

7 These results contrast with the findings in developed countries, such as the United States, where large firms tend to have lower VAX.
through SME and FIE, suggesting that although SOE’s direct participation in exports has been small, its actual participation and impact on China’s exports have been more important but overlooked. Similar to SOE, LP and SME both have a large share of indirect VAX in total VAX, though LP have a much lower VAX ratio. Compared to all firm types, FIE in China tend to export more directly.

While our paper is methodological in nature, we exploit the data generated to analyze the reasons behind the high and rising indirect export participation for both SOE and SME. Turning to the industry composition of indirect exports by firm type, we find that SOE’s indirect exports are due to their prevalence in upstream or non-tradable industries, such as energy and mining; metal and non-metallic mineral extraction; electricity; gas and water supply; and the financial sector. This may not be surprising since large domestic private firms also appear to have high indirect export shares in similar industries. While this prevalence of large firms in upstream industries could also be found in other countries, what we intend to show is that SOE, not only large firms, have been dominating the upstream of the domestic segment of GVC in China. Based on information from the IO tables for 2007 and 2010, we find evidence of significant increases in SOE’s VAX ratio, share of indirect VAX in total VAX, and share of VAX in aggregate exports. These estimated changes are statistically significant. The systematic documentation of this special pattern can offer important insights for understanding China’s past and future economic growth, and the political economic factors that shaped it.  

Our paper makes several contributions to the literature. First, it adds to the growing literature on measuring the extent of production fragmentation across national borders (e.g., Hummels, Ishii, and Yi, 2001, Johnson and Noguera, 2012a, 2012b; Koopman, Wang, and Wei, 2012; Koopman, Wang, and Wei, 2014). The focus of that literature has been on the relative shares of domestic versus foreign value added in international trade. The composition and dynamics of the domestic segment of GVC have not been subject to the same level of scrutiny. In particular, understanding how trade liberalization affects intra-national trade between industries and in turn shapes the reallocation of resources and across industries and firms is important for designing development policies. Our paper takes a first step by analyzing intra-national trade between different firm types, focusing on the roles of SOE and SME in China.

Related to the value-added trade literature, our approach extends the IO-table based approach to

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8 For instance, to the extent that SOE are less productive than non-state firms (e.g., Zhu, 2012), a deeper privatization of SOE or lower entry barriers in upstream industries may increase the efficiency of direct exporters in the downstream, which in turn increases the speed of upgrading of Chinese exporters’ along GVC. The conventional view is that China’s export growth is largely driven by the dynamic labor-intensive private sector, especially the foreign-dominated processing trade sector. Our findings add to this conventional view by showing that SOE, through their protected position in the upstream, have been playing an important role in shaping Chinese export patterns and performance.
incorporate the recent trade literature that emphasizes firm heterogeneity in international trade. In reality, firms differ substantially in their export intensity, import intensity, and their position and participation in GVC. Other characteristics such as ownership structure (domestic/foreign, private/public), location, size can also directly affect the way firms respond to trade liberalization and other economic shocks. The usual method that relies on the aggregate IO tables ignores most of the underlying firm heterogeneity. The lack of information on between-firm transactions in the micro data also restricts the construction of IO tables by firm type. Moreover, a widely recognized drawback of using IO tables to measure VAX is the assumption that firms within an industry use the same technology for production. Proportionality assumptions are often made in order to distribute imports into different final uses and different source countries, as information on bilateral trade between suppliers and users is generally not available at the country-industry level. Relatedly, Kee and Tang (2015) show that a country’s domestic content in exports computed using IO tables are generally biased downward. It is because larger firms, which statistical agencies rely on in constructing IO tables, tend to have higher import intensities. Our paper provides a method to reduce these measurement biases due to heterogeneity in export and import intensities across firm sizes and ownership types.

Our paper also contributes to the literature on the determinants of firm export participation and other indirect export channels. Research in international trade shows that only a small fraction of enterprises, usually the large ones, directly participate in international trade (e.g., Bernard et al., 2007, 2015). The standard argument is that exporting is typically associated with high fixed costs and only large (productive) firms can make sufficient export profits to justify such fixed costs. However, many non-exporters may engage in international trade indirectly, through intermediaries and by providing intermediate inputs and services to exporters, particularly large multinationals. While the first channel has received a lot of attention in the recent literature (e.g., Bernard et al., 2010 and Ahn, Khandelwal, and Wei, 2012), the second channel has not received the deserved attention, partly due to the lack of data on inter-firm transactions within a country. Our paper provides a method that combines firm-

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9 This literature started with Bernard et al. (2003) and Melitz (2003). See Bernard et al. (2007, 2015) for a comprehensive review of both the theoretical and empirical literatures on firms and trade.
10 These assumptions have been shown to lead to substantial biases in the estimation of countries’ value added, factor content of trade, and our general inference of the impact of trade on countries’ macro-economy (e.g., Puzzello, 2012). For instance, De La Cruz et al. (2011) and Koopman, Wang and Wei (2012) show that by allowing different imported material intensities for processing and non-processing exporters, the estimated foreign value added ratio in aggregate exports from both China and Mexico increases significantly.
11 As Bernard et al. (2007) described “engaging in international trade is an exceedingly rare activity: of the 5.5 million firms operating in the United States in 2000, just 4 percent were exporters. Among these exporting firms, the top 10 percent accounted for 96 percent of total U.S. exports.”
12 A notable exception is the report by the USITC (2010), who also uses the constrained optimization methodology to estimate the contribution of small and medium enterprise (SME) to US exports. The report finds that SME’s total contribution to U.S. exports increased from less than 28% to 41% in 2007, when the value of intermediates supplied by SME to exporting firms is taken into account.
level and industry-level data to quantify the volume of indirect exports, and through which channel “non-exporters” export indirectly.

Finally, our paper relates to previous attempts to cast the estimation of unknown values in IO tables as a constrained matrix balancing problem (van der Ploeg, 1988, Nagurney and Robinson, 1989, Bartholdy, 1991). It also contributes to the information theory literature that estimates interregional transactions using regional economic statistics and input-output accounts (Batten, 1982; Batten and Martellato, 1985, Canning and Wang, 2005). In particular, our paper is closely related to Golan, Judge and Robinson (1994), who also pose the estimation as an optimization problem with a nonlinear criterion objective function and multiple linear constraints.

The rest of this paper is organized as follows. Section 2 develops the conceptual model for our estimation. Section 3 introduces the quadratic optimization model. Section 4 explains how standard bootstrapping can be combined with our method to compute standard errors of our estimates. Section 5 describes the data source and how to initialize the optimization. Section 6 reports the estimated VAX for different firm types in China. Section 7 concludes.

2. Conceptual Model

This section develops a model to extend a standard IO table into a more elaborate account one that tracks domestic inter-sector transactions between different firm types. It defines the concepts of direct and indirect value added exports (VAX), and shows how to decompose indirect exports into their different domestic input-output channels based on firm types. It also specifies which variables cannot be readily computed using standard IO tables and thus need to be estimated.

The standard IO table reports information on sales of intermediate inputs by one industry to another in the domestic economy. By construction, summing up elements horizontally across each row and vertically across each column will both yield the same gross output of an industry.13 To study the intra-national trade between different types of firms based on their ownership types and sizes, we first split the non-competitive IO table into numerous sub-accounts based on the firm characteristics of interest. Since we will implement our estimation using Chinese data, to simplify the discussion, let us consider splitting the 42-sector non-competitive IO table of China into 6 sub-accounts,14 based on 3

13 The vertical summation is analogous to the production approach of measuring a country’s gross product (GNP), which decomposes gross output into payments to different intermediate inputs and primary factors of production. The horizontal summation is analogous to the expenditure approach of measuring a country’s GNP, which decomposes an industry’s gross output into its various types of domestic absorption as well as exports.
14 The non-competitive IO table assumes that imported and domestic products are not substitutable, in contrast to the standard IO table that assumes perfect substitutability between imported and domestic products. When competitive IO
ownership types: State (SOE), Foreign (FIE), or Private (P); and 2 size categories: large and small. Since there are altogether 252 groups (42 industries × 3 ownership types × 2 sizes), we need to estimate 252 × 252 (including the within-group transactions) unknown values of domestic transactions between any pair of firm types. Fig. 1 illustrates the extended IO table. From now on, matrices and vectors will be presented in boldface.

(Insert Figure 1 here)

In Fig. 1, Z, Y, E, X, and M represent, respectively, intermediate inputs, domestic final demand, exports, total output, and imports. We use a two-alphabet superscript to denote one of the 6 firm groups. The first alphabet denotes ownership type (SOE, FIE, or P) while the second subscript denotes size (L or S). A combination of a size category and an ownership type gives us a firm group, g. Specifically, g can be SL, SS, FL, FS, PL, or PS, which stand for Large SOE, Small SOE, Large FIE, Small FIE, Large, and Small Private Firms, respectively. Subscripts i and j are for supplying and buying product categories (42 of them), which will be mostly referred to as sectors from now on.

The last three rows in Fig. 1 report imported intermediate inputs, value added and the column sum of gross output, respectively. The last three columns are respectively domestic final use, exports, and total gross output. The remaining part of the matrix is a 6×6 block of square matrices, each of which is 42×42 in dimension. For example, \(Z^{SL,SL}\) in the first row (SL) and first column (SL) is a 42×42 matrix, with an element in row i and column j, \(z_{ij}^{SL,SL}\), representing output produced by LSOE in sector i used as intermediate inputs by other LSOE in sector j. Moving horizontally across the first row, each matrix, \(Z^{SL,g}\), is a 42×42 matrix with an element \(z_{ij}^{SL,g}\) in row i and column j representing output that is still produced by LSOE in sector i but is used as intermediate inputs by group-g firms in sector j. Similarly, when moving down vertically within a column, each entry is a 42×42 matrix, \(Z^{g1,g2}\), with elements, \(z_{ij}^{g1,g2}\), being the output produced by firms in group g1 and sector i, and used as intermediate inputs by firms in group g2 and sector j.

Moving to the last three rows of the extended IO table, the first 6 entries in row 7 (F) are 42×42 matrices, \(Z^{F,g2}\). The element in row i and column j of \(Z^{F,g2}\), \(z_{ij}^{F,g2}\), represents product i imports that are used as intermediate inputs by group-g2 firms in sector j. The 7th entry, \(Y^F\), is a 42×1 vector, with

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Tables are used, only one set IO coefficients are needed. The underlying Leontief or linear production functions assumed in either approach have their obvious drawbacks, but we consider our approach, which permits different IO coefficients on imported and domestic inputs across sector-pairs, to be more suitable for the purpose of our study.
element, $y_i^F$, being product $i$ imports for final consumption. The last entry in row 7, $M$, is a 42×1 vector, with element $m_i$ representing total imports of product $i$. By definition, $m_i$ is the sum of the first 7 entries in the same row. Rows 8 and 9 show sectoral value added and gross output of the 6 different firm groups, respectively. For example, in the first column in Row 8, $V^{SL}$ is a 1×42 row vector that has element $i$ equal to the direct value added of LSOE in sector $i$ (cost of production factors). In the last row, $(X^{SL})^T$ is a 1×42 row vector with element $i$ being the gross output of LSOE in sector $i$. Superscript $T$ represents the transpose operation. Other $X$ and $V$ matrices are defined similarly for different firm groups.

The input-output coefficients in the extended IO table can be expressed in matrix algebra as:

$$A^{g1,g2} = \begin{bmatrix} a_{ij}^{g1,g2} \\ z_{ij}^{g1,g2} \\ x_j^{g2} \end{bmatrix}$$

and

$$A^{F,g2} = \begin{bmatrix} a_{ij}^{F,g2} \\ z_{ij}^{F,g2} \\ x_j^{g2} \end{bmatrix},$$

where $i$ is the row subscript and $j$ is the column subscript. $A^{g1,g2}$ is a 42×42 block matrix, with each element being an IO coefficient representing the amount of output produced by firms in group $g1$ used as intermediate inputs in the production of one unit of output by group-$g2$ firms. $g1$ and $g2$ can each be one of the six firm types. More specifically, $x_j^{g2}$ represents output by group-$g2$ firms in sector $j$. It is also the $j$th element in $(X^{g2})^T$ in the last row of Fig. 1. $z_{ij}^{g1,g2}$ represents sector $i$ output produced by group-$g1$ firms that are used by group-$g2$ firms in sector $j$. It is the element in row $i$ and column $j$ of $Z_j^{g1,g2}$.

Similarly, $A^{F,g2}$ is a 42×42 matrix, with each element being an IO coefficient measuring the amount of imported goods used as intermediate inputs by group-$g2$ firms to produce one unit of gross output. In other words, the element in row $i$ and column $j$ of $Z_j^{F,g2}$ in the 3rd row from the bottom of Fig. 1, $z_{ij}^{F,g2}$, represents sector-$i$ imports used by group-$g2$ firms in sector $j$.

Let us define matrix $A$, which has 294 (7×42) rows and 252 (6×42) columns, as the IO transaction matrix:

$$A = \begin{bmatrix} A^d \\ - \\ - \\ A^m \end{bmatrix}$$

where
\[ \mathbf{A}^d = \begin{bmatrix} \mathbf{A}^{SL,SL} & \mathbf{A}^{SL,SS} & \mathbf{A}^{SL,FL} & \mathbf{A}^{SL,FS} & \mathbf{A}^{SL,PL} & \mathbf{A}^{SL,PS} \\ \mathbf{A}^{SS,SL} & \mathbf{A}^{SS,SS} & \mathbf{A}^{SS,FL} & \mathbf{A}^{SS,FS} & \mathbf{A}^{SS,PL} & \mathbf{A}^{SS,PS} \\ \mathbf{A}^{FL,SL} & \mathbf{A}^{FL,SS} & \mathbf{A}^{FL,FL} & \mathbf{A}^{FL,FS} & \mathbf{A}^{FL,PL} & \mathbf{A}^{FL,PS} \\ \mathbf{A}^{FS,SL} & \mathbf{A}^{FS,SS} & \mathbf{A}^{FS,FL} & \mathbf{A}^{FS,FS} & \mathbf{A}^{FS,PL} & \mathbf{A}^{FS,PS} \\ \mathbf{A}^{PL,SL} & \mathbf{A}^{PL,SS} & \mathbf{A}^{PL,FL} & \mathbf{A}^{PL,FS} & \mathbf{A}^{PL,PL} & \mathbf{A}^{PL,PS} \\ \mathbf{A}^{PS,SL} & \mathbf{A}^{PS,SS} & \mathbf{A}^{PS,FL} & \mathbf{A}^{PS,FS} & \mathbf{A}^{PS,PL} & \mathbf{A}^{PS,PS} \end{bmatrix}, \]

and \( \mathbf{A}^m = [\mathbf{A}^{F,SL} \quad \mathbf{A}^{F,SS} \quad \mathbf{A}^{F,FL} \quad \mathbf{A}^{F,FS} \quad \mathbf{A}^{F,PL} \quad \mathbf{A}^{F,PS}] \).

Let us also define \( \mathbf{A}_g^{g1} = \begin{bmatrix} v_{g1}^j \\ v_{g1}^j \end{bmatrix} \) as the value added coefficient vector (1 by 42) for firm group \( g1 \) where \( v_{g1}^j \) is the \( j \)th element of \( \mathbf{V}^{g1} \) in the second last row in Fig. 1; and \( \mathbf{A}_V = [\mathbf{A}^{SL}_V, \mathbf{A}^{SS}_V, \mathbf{A}^{FL}_V, \mathbf{A}^{FS}_V, \mathbf{A}^{PL}_V, \mathbf{A}^{PS}_V] \) as the 1×252 row vector of value added, covering all sectors and firm groups.

Because total gross output \( (x) \) in any sector has to be equal to direct value-added \( (v) \) plus the cost of domestic intermediate inputs \( (z) \) from all firm types and imported inputs \( (z^f) \), the following accounting identity always holds:

\[ \mathbf{u} = \mathbf{A}_V + \mathbf{uA}^d + \mathbf{A}^m, \quad (1) \]

which means that each unit of output can be attributed to direct value added, domestic intermediate inputs, and imported intermediate inputs. \( \mathbf{u} \) is a 1×252 row vector and \( \mathbf{A}^m \) is a 1×42 row vector, respectively.

Taking \( \mathbf{uA}^d \) to the left hand side of eq. (1) and rearranging it yields

\[ \mathbf{u} = \mathbf{A}_V(\mathbf{I} - \mathbf{A}^d)^{-1} + \mathbf{A}^m(\mathbf{I} - \mathbf{A}^d)^{-1} = \mathbf{A}_V \mathbf{B} + \mathbf{A}^m \mathbf{B}, \quad (2) \]

where \( \mathbf{B} = (\mathbf{I} - \mathbf{A}^d)^{-1} \) is the well-known Leontief matrix. The intuition behind the Leontief matrix is as follows: for each dollar of exports, the first round of value added generated by the direct exporters is what we call direct VAX. To produce direct VAX, intermediate inputs have to be used, which in turn generate additional value added, and so on. Such a process of value-added generation continues

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\(^{15}\) Similar to \( \mathbf{A} \), \( \mathbf{B} \) is a high dimensional matrix that is composed of 6 x 6 block matrices. Each block matrix, \( \mathbf{B}^{g1,g2} \), is a 42×42 matrix with elements equal to the total requirement coefficients, representing the amount of required gross output by firm group \( g1 \) for a one unit increase in domestic final demand or exports.
iteratively and can be traced throughout the domestic input-output linkage across firm types and sectors in the economy. The total VAX induced by one dollar of exports is thus equal to the sum of direct and all rounds of indirect VAX generated.

Post-multiplying both sides of eq. (2) by the diagonal matrix of exports $\mathbf{E}$, yields

$$\mathbf{uE} = A_V \mathbf{B} \mathbf{E} + \mathbf{0} \mathbf{A}^m \mathbf{B} \mathbf{E}, \quad \text{(3)}$$

Notice that $A_V = \mathbf{uA}_V$, where $\mathbf{A}_V$ is the diagonal matrix of $A_V$ with the dimension of $252 \times 252$. Thus, eq. (3) can be rewritten as

$$\mathbf{uE} = \mathbf{uA}_V \mathbf{B} \mathbf{E} + \mathbf{0} \mathbf{A}^m \mathbf{B} \mathbf{E}. \quad \text{(4)}$$

Eq. (4) states that the country's total gross export value $\mathbf{uE}$, a $1 \times 252$ row vector, can be decomposed into VAX in exports $\mathbf{uA}_V \mathbf{B} \mathbf{E}$ (either used directly for production of exported goods and services, or indirectly by firms that supply domestic inputs that are used by exporters), and imported materials embedded in exports $\mathbf{0} \mathbf{A}^m \mathbf{B} \mathbf{E}$, which includes imported intermediates used directly by exporters or embodied in other domestic intermediates used by them.

$\mathbf{uA}_V \mathbf{B} \mathbf{E}$, the first term on the right hand side of eq. (4), is the key to our quantification of VAX. Specifically, $\mathbf{A}_V \mathbf{B} \mathbf{E}$ is a $252 \times 252$ square matrix, with each element representing the source (from which sector and firm type) and the channel (indirectly used by which sector and firm type) of VAX. Depending on the research question, one can aggregate $\mathbf{A}_V \mathbf{B} \mathbf{E}$ vertically or horizontally to estimate VAX. If the goal is to decompose VAX to how and where it is used by downstream firm-sector types, we should use the forward-linkage approach by summing up the elements of $\mathbf{A}_V \mathbf{B} \mathbf{E}$ horizontally across each row. If the goal is to measure VAX embodied in gross exports by firm type based on their source of contribution by sector-firm-type, regardless of the sector or firm-type in which the value added is originally created, we should use the backward-linkage approach by summing up the elements of $\mathbf{A}_V \mathbf{B} \mathbf{E}$ vertically along each column.\(^ {16} \) Put it differently, we will first use the forward-linkage approach to examine how VAX by firm type are generated throughout the entire domestic production network. Then we will use the backward-linkage approach to examine how each downstream firm-type and sector’s gross exports can be sourced back to each of the sector-firm-type’s upstream value-added origins.\(^ {16} \)

\(^{16}\) See Wang, Wei and Zhu (2013) for a more detailed discussion on forward- and backward-linkage approaches to measure value-added exports.
Since we are interested in both direct and indirect VAX, we decompose the Leontief matrix $B$ and rewrite the 252×252 $VAX$ matrix as

$$VAX = \hat{A}_v B \hat{E} = \hat{A}_v \hat{E} + \hat{A}_v (B - I) \hat{E}. \quad (5)$$

On the right hand side of eq. (5), the first term, $\hat{A}_v \hat{E}$, captures direct VAX, while the second term, $\hat{A}_v (B - I) \hat{E}$, represents indirect VAX. We can further decompose $\hat{A}_v (B - I) \hat{E}$ into indirect VAX via other firms within the same firm group (e.g. SOE exporting via other SOE) or via other firm groups (e.g., SOE exporting via FIE). The same-group indirect exports can be derived from the multiples that include only the diagonal of $\hat{A}_v (B - I) \hat{E}$. The between-group indirect exports can be derived from the multiples including only the off-diagonal part of $\hat{A}_v (B - I) \hat{E}$.

To implement the forward-linkage approach so that we can trace the final use of value added created by the primary factors employed in a particular sector-firm-type, we post-multiply both sides of eq. (5) by a 252×1 unit column vector, $\mu$. This operation essentially sums up each sector-firm-type’s value added horizontally to obtain a measure of VAX in exports at the sector-firm-type level, regardless of which downstream sector-firm-type’s exports such value-added is embedded in. Formally, the forward-linkage based VAX in exports is

$$VAX_{fw} = VAX \mu = \hat{A}_v \hat{E} \mu + \hat{A}_v (B - I) \hat{E} \mu, \quad (6)$$

where $VAX_{fw}$ is a 252×1 column vector.

Eq. (6) can be further decomposed along the firm-type dimension. The first row in $\hat{A}_v \hat{E} \mu$ represents the direct VAX from large SOE (SL). The first row of the second term, $\hat{A}_v (B - I) \hat{E} \mu$, is the sum of 6 multiples as follows:

$$\hat{A}_v^SL (B^{SL,SL} - I) \hat{E}^{SL} \hat{\mu} + \hat{A}_v^SL B^{SL,SS} \hat{E}^{SS} \hat{\mu} + \hat{A}_v^SL B^{SL,FL} \hat{E}^{FL} \hat{\mu}$$

$$+ \hat{A}_v^SL B^{SL,FS} \hat{E}^{FS} \hat{\mu} + \hat{A}_v^SL B^{SL,PL} \hat{E}^{PL} \hat{\mu} + \hat{A}_v^SL B^{SL,PS} \hat{E}^{PS} \hat{\mu}, \quad (7)$$

where $\hat{\mu}$ is a 42×1 column vector. $\hat{A}_v^SL (B^{SL,SL} - I) \hat{E}^{SL} \hat{\mu}$ is indirect VAX via large SOE firms, $\hat{A}_v^SL B^{SL,SS} \hat{E}^{SS} \hat{\mu}$, $\hat{A}_v^SL B^{SL,FL} \hat{E}^{FL} \hat{\mu}$, $\hat{A}_v^SL B^{SL,FS} \hat{E}^{FS} \hat{\mu}$, $\hat{A}_v^SL B^{SL,PL} \hat{E}^{PL} \hat{\mu}$, and $\hat{A}_v^SL B^{SL,PS} \hat{E}^{PS} \hat{\mu}$ represent LSOE’s indirect VAX via SSOE, LFIE, SFIE, LP, and SME’s exports, respectively. Other rows in eq. (6) can be interpreted similarly for other firm types. Eq. (6) thus provides detailed information about the volume of direct and indirect VAX, as well as through what types of firms that indirect exporting
takes place. If we consider the 42 sectors within each firm-group-sector-pair, we can analyze these different components of VAX by sector.

To implement the backward-linkage approach that decomposes each firm type’s gross exports into their original value-added source by sector and firm-type, we pre-multiply both sides of eq. (5) by the 1×252 unit row vector \( u \). This operation essentially sums up each sector-firm-type’s VA vertically to obtain a measure of VAX at the sector-firm-type level. Formally, the backward-linkage based VAX in exports is

\[
VAX_{bw} = uVAX = u\hat{A}_V\bar{E} + u\hat{A}_V(B - I)\bar{E}. \tag{8}
\]

By replacing \( u\hat{A}_V\bar{B}\bar{E} \) in eq. (8) by eq. (4), we can completely decompose China’s gross exports according to its various VAX sources as follows:

\[
u\bar{E} = u\hat{A}_V\bar{E} + u\hat{A}_V(B - I)\bar{E} + \theta A^mB\bar{E}. \tag{9}
\]

Notice that all terms in eq. (9) are 1×252 row vectors.

The first column of the first term, \( u\hat{A}_V\bar{E} \), represents the direct VAX by large SOE (SL) in all 42 sectors. Notice the direct VAX based on the forward-linkage and backward-linkage approaches are identical (i.e. \( (u\hat{A}_V\bar{E})^T \) in eq. (9) = \( \hat{A}_V\bar{E}u \) in eq. (4)). However, the indirect value-added exports measures can be very different for each firm group-sector pair. The two measures are only equal to each other at the country level (see Wang, Wei, and Zhu, 2013 for details). In the second term, \( u\hat{A}_V(B - I)\bar{E} \), the first column is the sum of 6 multiples as

\[
\bar{u}\hat{A}_V(B_{SL,SL} - I)\bar{E}_{SL} + \bar{u}\hat{A}_V^SSB_{SS,SL}\bar{E}_{SL} + \bar{u}\hat{A}_V^FB_{FL,SL}\bar{E}_{SL} \\
+ \bar{u}\hat{A}_V^FSB_{FS,SL}\bar{E}_{SL} + \bar{u}\hat{A}_V^PLB_{PL,SL}\bar{E}_{SL} + \bar{u}\bar{B}_{PS,SL}\bar{E}_{SL} \tag{10}
\]

where \( \bar{u} \) is a 1×42 row vector. \( \bar{u}\hat{A}_V(B_{SL,SL} - I)\bar{E}_{SL} \) is LSOE’s indirect VAX via large LSOE; \( \bar{u}\hat{A}_V^SSB_{SS,SL} \), \( \bar{u}\hat{A}_V^FB_{FL,SL} \), \( \bar{u}\hat{A}_V^FSB_{FS,SL} \), \( \bar{u}\hat{A}_V^PLB_{PL,SL} \), and \( \bar{u}\hat{A}_V^{PS,SL} \) represent SSOE, LFIE, SFIE, LP, and SME’s value-added embodied in LSOE’s gross exports, or these firm groups’ indirect VAX via LSOE, respectively. Other columns of \( u\hat{A}_V(B - I)\bar{E} \) in eq. (9) can be interpreted similarly for other firm groups. Therefore, eq. (10) provides detailed information about the sources of VAX produced by each firm group. By considering all 42 sectors within each firm-group-
pair, we can analyze the value-added composition for each firm group by sector.\footnote{The full decomposition of each firm type’s exports by value-added sourced from the 6 firm groups and 42 sectors are available upon request.}

3. Estimation Method

Eqs. (5) through (10) can be used to study the indirect value added by firm type at the aggregate and sector levels, and decompose each firm type’s sectoral exports into its various VAX sources. However, statistical agencies in most countries usually provide only the conventional IO matrix, $A$, and not the disaggregated block matrices by firm groups, such as $A_{g1.g2}$ or $A^{F.g2}$. Thus, we need to develop methods to estimate these subaccounts.

Before describing our estimation methods, let us revisit what information a typical IO table provides. At the sector level, a typical national IO table contains the following information:

\begin{itemize}
  \item $x_i$: gross output of sector $i$;
  \item $z_{ij}^D$: domestic goods from sector $i$ used as intermediate inputs in sector $j$;
  \item $z_{ij}^F$: imported goods from sector $i$ used as intermediate inputs in sector $j$;
  \item $v_j$: value added in sector $j$;
  \item $e_i$: total exports of sector $i$ goods;
  \item $m_i$: total imports of sector $i$ goods;
  \item $y_i^D$: total domestic final-good demand for sector $i$ goods (excluding exports);
  \item $y_i^F$: total final-good demand for imported goods $i$.
\end{itemize}

These data from the standard IO table provide the adding up constraints for our optimization. They restrict the estimated values of our extended IO table to be always added up back to the values in the original IO table. To estimate our extended table with 6 sub-accounts, we complement the aggregate data with official firm-level data (See Section 4 for details).

The key unknown to be estimated are the inter-sector transaction flows among different firm types, (i.e., $[z_{ij}^{g1.g2}]$ for each $g1$ and $g2$, where $g1$ and $g2$ belong to one of the six firm types. We also need to estimate the use of imported intermediate input supplied by sector $i$ and purchased by each firm type $g$ in sector $j$ (i.e., $[z_{ij}^{F.g}]$ ). Finally, we also need to estimate sector-level domestic final demand by firm type $g$, $[y_{ij}^{g}]$, which are typically not available from a standard IO table but can be constructed using firm-level data.

To estimate these values, we develop a quadratic optimization model that uses information from standard national IO tables, sector-level statistics, and firm-level data. The optimization model has the following objective (penalty) function:
Min \( S = \sum_{g1=SL}^{OS} \sum_{g2=SL}^{OS} \left( \sum_{i=1}^{K} \sum_{j=1}^{K} \frac{(z_{ij}^{g1,g2} - z_{0ij}^{g1,g2})^2}{z_{0ij}^{g2}} \right) \)

\( + \sum_{g=SL}^{OS} \left( \sum_{i=1}^{K} \sum_{j=1}^{K} \left( \frac{z_{ij}^{Fg} - z_{0ij}^{Fg}}{z_{0ij}^{Fg}} \right)^2 \right) + \sum_{g=SL}^{OS} \left( \sum_{j=1}^{K} \frac{(y_{ij}^{g} - y_{0ij}^{g})^2}{y_{0ij}^{g}} \right) \)  \( (11) \)

Importantly, the solutions to the above optimization need to satisfy the following six groups of linear constraints:

\[ \sum_{g2=SL}^{OS} \sum_{j=1}^{K} \left( \frac{z_{ij}^{g1,g2}}{z_{0ij}^{g2}} \right) + \sum_{i=1}^{K} \sum_{j=1}^{K} \left( \frac{z_{ij}^{Fg}}{z_{0ij}^{Fg}} \right)^2 \]  \( (12) \)

\[ \sum_{g1=SL}^{OS} \sum_{i=1}^{K} \sum_{j=1}^{K} \left( \frac{z_{ij}^{g1,g2}}{z_{0ij}^{g2}} \right) + \sum_{i=1}^{K} \sum_{j=1}^{K} \left( \frac{z_{ij}^{Fg}}{z_{0ij}^{Fg}} \right)^2 \]  \( (13) \)

\[ \sum_{g1=SL}^{OS} \sum_{g2=SL}^{OS} z_{ij}^{g1,g2} = z_{ij}^{D} \]  \( (14) \)

\[ \sum_{g=SL}^{OS} z_{ij}^{Fg} = z_{ij}^{F} \]  \( (15) \)

\[ \sum_{g=SL}^{OS} y_{ij}^{g} = y_{ij}^{D} \]  \( (16) \)

\[ \sum_{g=SL}^{OS} \sum_{j=1}^{K} z_{ij}^{Fg} + y_{ij}^{F} = m_{ij} \]  \( (17) \)

the following non-negativity constraints:

\[ z_{ij}^{g1,g2} \geq 0; \quad z_{ij}^{Fg} \geq 0; \quad y_{ij}^{g} \geq 0, \]  \( (18) \)

and the following adding-up constraints:

\[ \sum_{g=SL}^{OS} \frac{v_{ij}^{g}}{v_{ij}^{g}} = v_{ij}; \quad \sum_{g=SL}^{OS} \frac{x_{ij}^{g}}{x_{ij}^{g}} = x_{ij}; \quad \sum_{g=SL}^{OS} \frac{e_{ij}^{g}}{e_{ij}^{g}} = e_{ij}. \]  \( (19) \)

In the objective function (11), the target variables that we aim to estimate are indicated with \( \hat{} \) while initial values for these targets are indicated with 0. To kick-start the constrained optimization, we set initial values for all these unknown variables based on various proportionality assumptions and micro data from Chinese official sources, which will be discussed in detail in Section 5. Notice that the inverse of the initial values are used as the weights in the objective function to reduce the penalty for large deviations for large values according to the data (e.g., basic business services tend to have a higher cost share for many sectors). We have conducted sensitivity analysis by using different initial
values. It turns out that our results are not sensitive to using different initial values.\(^{18}\)

Depending on the reliability weights chosen, the quadratic optimization model covers a broad range of commonly used linear estimators. If the weights are all equal to one, the model resembles a constrained least squares estimator. If initial values are used as weights as what we do in this paper, the model resembles a weighted constrained least square estimator. If the weights are set proportional to the variances of the initial values, and if the initial values are statistically independent, the model yields unbiased linear estimates of the true unknown variables (Byron, 1978). If the weights are set exactly equal to the variances of the initial values (Stone, 1984, van der Ploeg, 1988), the model will be identical to the Generalized Least Squares estimator. Finally, as noted by Stone et al. (1942) and proven by Weale (1985), when the errors of the initial values are normally distributed, the solutions satisfy the maximum likelihood criteria.

In the linear constraints (eqs. (12) through (17)), aggregate statistics are kept constant throughout the optimization process. There are two data sources from which we obtain these constants. The first source is the firm data, which we use to compute total gross output \((x^g_i)\), exports \((e^g_i)\), and value added \((v^g_i)\) by each firm type in sector \(i\). These variables are indicated with \(\bar{\text{-}}\). We can compute standard errors for these constants using bootstrapped firm samples (see Section 4 below). The second source is the IO table, from which we obtain information on domestic goods from sector \(i\) used as intermediate inputs in sector \(j\) \((z^D_{ij})\), imported goods from sector \(i\) used as intermediate inputs in sector \(j\) \((z^F_{ij})\), total imports of sector \(i\) goods \((m_i)\), total domestic final demand for sector \(i\) goods \((y^D_i)\), and final-good demand for imported goods \(i\) \((y^F_i)\). Not only that these constants from IO tables are kept constant through the optimization process, they are also constant across bootstrapped samples.

All constraints need to be satisfied for all \(i\) (42 of them) and \(j\) (42 of them), \(g\) (6 of them), \(g1\) (6 of them), and \(g2\) (6 of them). These constraints have straightforward economic interpretations. Eq. (12) is a set of supply-and-use balancing (row sum) constraints for the extended IO table. It states that total gross output by each type of firms in sector \(i\) must equal the sum of their use of intermediate inputs, exports, and supply of goods and services to final domestic consumers. Eq. (13) is the set of production and cost balancing (column sum) constraints. It defines the value of gross output by each type of firms in sector \(j\) as the sum of intermediate inputs and primary factors used in the production process. Eqs. (14) to (17) are a set of adding up constraints to ensure that the solutions from the model sum up to the aggregate statistics (i.e., domestic final demand, imports, and inter-sector transactions) in the official

\(^{18}\) Because our model features a concave objective function and linear constants, the model solutions are restricted into a convex set, which will be relatively stable with respect to variations in the initial values as long as all parameters in the constraints are kept constants. This is a well-known property of a linear estimator, such as the ordinary least square estimator.
IO table at the sector and sector-pair levels. It is important to note that the initial values we set are unlikely to satisfy any of these linear restrictions of the model.

Our estimation model is flexible enough to take into account a wide range of information in the optimization process. Additional constraints, such as upper and lower bounds imposed on unknown variables, can be added. Extra terms in the objective function to penalize deviations of solutions from select linear constraints can also be added. Such flexibility is particularly important for obtaining optimal solutions when there are inconsistencies in the linear constraints, which could arise partly due to the use of different data sources.

4. Computing Standard Errors For the Extended Tables

Any estimation, by definition, must be associated with measurement errors. Although we confirm that the initial values we set play a insignificant role in determining the final estimates, one may be particularly concerned about how our estimates are sensitive to the linear constraints we impose in our optimization. In this section, we discuss how to incorporate the standard bootstrapping procedures with our optimization model to obtain standard errors of the estimates.

It is worth noting that developing a method to compute standard errors of our estimates has a wider appeal beyond the current context. One such application is to assess the accuracy of any national IO table. IO tables provided by statistical agencies are survey-based and thus contain measurement errors. Some of them are due to errors of reporting, while others are due to assumptions made by researchers in the absence of crucial information. A classic example includes different kinds of proportionality assumptions, which are often made when information about how imported inputs from a sector were allocated to different users are unknown. Our method of constructing standard errors can be used not only to assess the accuracy of our estimation, but also to gauge the accuracy of the coefficients of any IO tables provided by statistical agencies, as long as the corresponding micro data are available to measure standard errors.

Our proposed procedures of obtaining standard errors follow closely the standard bootstrapping procedure. Micro-level data corresponding to the dimensions we extend the IO table are required. The main idea is to create many random samples of extended tables, and use them to construct sample distributions of the estimated IO transaction flows. Based on the distribution of the estimates, we then

19 See Lenzen et al. (2010) for various reasons for why the numbers reported by a standard IO table may contain measurement errors.
20 See Puzzello (2012) for an illustration of the potential biases in the measurement of domestic content, foreign content, and factor content in trade, due to the proportionality assumptions made about imported input usage.
compute their standard errors and confidence intervals of all estimated values in the extended IO table. Specifically, we use information on firms’ total sales, value-added, exports, employment, and ownership types from the 2008 census data. Within each firm-type-sector group (6 firm types x 42 broad sectors), we randomly draw firms with replacement. The number of draws in each group is set equal to the actual number of firms in the group according to the census data. By using each random sample, we compute gross output, export, wages, and surplus for each of the 252 firm type-sector group. We then use the data computed from each bootstrapped sample to set the constants in the linear constraints (eq. (12)-(17) above) and initial values in the objective function (11) in the optimization model to estimate a new extended IO table. We then repeat the bootstrapping and optimization exercises until 2000 extended IO tables are estimated.

With 2000 extended IO tables, we can now construct a distribution of each estimate in the extended IO table. Overall, the magnitudes of the standard errors of the IO coefficients, compared to the estimated IO table coefficients, are relatively small. Most of them are within 10% of the coefficient estimates. There are a few exceptions in which the standard errors are large. When reporting our results below, we will provide the 95% confidence intervals of the estimates, whenever applicable. We will also report the standard-error-to-mean ratios of the values used in the linear constraints for different firm types to show how the large standard deviation of firm values within each firm type may lower the precision of our estimates.

5. Data Sources and Variable Initialization

As described in Section 3, the initial values and constants in the linear constraints of the model are computed using data from both firm-level data and IO tables. We implement the optimization using the 42-sector “non-competitive” IO tables for both 2007 and 2010, along with firm census data for

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21 Robinson et al. (2001) develop a method to handle measurement errors in cross-entropy minimization by using an error-in-variables formulation. Estimating the error variances in a large data set using their approach remains computationally challenging.

22 Important information to categorize firms is sometimes missing for some firm-sector groups. For those groups, we make the following data assumptions. We assume that all firms in the agricultural sectors are SME. Moreover, since the 2008 firm census data do not cover firms from the railroad and transportation sector, we use information from the 135-sector version of IO table to extend the 2007 and 2010 IO tables. In addition, we assume that all firms in the railroad sector are LSOE, while firms in other transportation sectors are assigned based on their size according to the 2008 firm census data. For service sector firms with zero export, we use a proportionality assumption to impose the share of exports.

23 Note that in our bootstrapping exercise, some IO tables generated cannot be used as some balancing conditions (i.e., eqs. (12)-(17)) are not satisfied. When initializing our quadratic optimization, we need to use aggregate statistics computed from the micro data to set the right hand sides of the balancing conditions (eqs. (12)-(17)). Since these statistics are known computed from random samples drawn from the firm census, sometimes they can take extreme values. Our quadratic optimization will fail to converge as one of the balancing conditions fails to hold. We discard those tables (less than 10%) and keep drawing until we have a sample of 2000 bootstrapped tables.

24 Results are available upon request.
2008 from China. Both data sources are from China’s National Bureau of Statistics (NBS). The firm census data cover over 5 million enterprises in China, including all state-owned and private enterprises from all manufacturing and non-manufacturing sectors. Balance sheet information, such as registration ownership type, equity share by ownership type, output, value added, four-digit industry code (about 900 categories), exports, employment, original value of fixed assets, and intermediate inputs. The ownership type of a firm in our analysis is defined based on the firm’s registration type or equity share by ownership. Specifically, a firm is considered state-owned (foreign) if it is registered as a state (foreign) company or has more than 50% equity owned by state (foreign) investors. The same criteria is used to define FIE and private firms. We will report estimates for both 2007 and 2010, but notice that all changes between the two years are due to changes in the IO tables, not from the census data as we only have one year of the latter.

For all sector pairs in the IO table, we aim to estimate transactions among any six sub-groups by ownership type and size: large SOE (LSOE), small and medium SOE (SSE), large FIE (LFIE), small and medium FIE (SFIE), large private enterprises (LP), and small and medium private enterprises (SME). Firm size category (large and small-and-medium) is determined by firm employment and sales, with thresholds specified by the NBS, with criteria varying across industries. Table A1 in the appendix reports those criteria.

The decision of putting firms into 6 groups is supported by the underlying firm distribution of export intensity and value added to sales ratios reported in the firm-level data. Fig. 2 shows that firm average export intensity differs significantly across ownership types, not so much along the firm size dimension. In particular, FIE are a lot more export-oriented than non-FIE firms. Fig. 3 shows that FIE also tend to have a higher value added to output ratio (VAY) than non-FIE firms. Within non-FIE firms, large firms tend to have higher VAY. Within FIE, there is little difference in these key variables between Hong Kong SAR, China, Macau, and Taiwan, China (HKMT) firms and non-Chinese FIE. Based on these findings, we separate firms based on 3 ownership types and 2 sizes, and group HKMT firms with other FIE. Putting firms into more refined categories comes with a cost of having too few firms in each cell and thus a less precise estimated VAX.

After assigning firms from the census to different groups, we use total sales/receipts at the group level to allocate gross output of each sector to each ownership-size type and value-added information to split wage and profit by firm type. We also assign exports (but not imports) to firm types in almost all industries based on our firm census data. Detailed import data, obtained from China’s Customs Administration, are disaggregated by firm ownership type within each 8-digit HS level. We use the United Nations Broad Economic Categories (BEC) code to separate intermediates from final goods in imports at the 6-digit-HS level, which are then aggregated up to 42 product categories in the Chinese
IO table. These data are used to split the total sector imports into firm type-sector pair and set the import-related constraints.

To initialize all $z_{ij}$’s in the objective function (11), we need to allocate each industry’s total intermediate inputs, both domestic and imported, into different product groups by firm type. To this end, we first use the NBS firm census and the original IO table to compute for each firm type (6 of them), the sectoral (42 sectors) output $x_{0j}^g$ and value added $v_{0j}^g$. Then we compute total intermediate inputs $(x_{0j}^g - v_{0j}^g)$ for each sector and firm type and the share of intermediate inputs of each firm type in sector $j$. Using these shares, we distribute the numbers $z_{0ij}^D$ and $z_{0ij}^F$ from the original IO table into 6 different firm types, e.g., $z_{0ij}^{g1,g2}$. Table A4 and A5 in the appendix report the shares of these variables by firm type in each of the 42 sectors. The specific procedures to set the initial values for the target variables in the optimization model are described below.

1. Setting the initial value for $z_{0ij}^{F,g}$ (the IO coefficients for imports for firm group $g$) involves two steps. For sectors that have zero imports of intermediate inputs in the customs trade statistics, but positive values in the IO table (such as various service sectors), we simply use the shares of each firm type in the sector’s total intermediate inputs and set the initial value for $z_{0ij}^{F,g}$ as

$$
z_{0ij}^{F,g} = \frac{x_{ij}^g - v_{ij}^g}{\sum_{g,j}(x_{ij}^g - v_{ij}^g)}z_{ij}^F, \quad (g = SL, SS, FL, FS, PL, PS) \tag{20}$$

On the other hand, for sectors that have positive imported intermediate inputs in the trade statistics, we first compute each firm group’s share in the sector’s imported inputs based on customs statistics to allocate imported inputs into SOE, FIE, and others. Using this adjusted $z_{ij}^F$ and eq. (20), we further allocate the imported inputs belonging to each ownership type to large and small-and-medium firms within the same ownership type, respectively.

2. To set the initial value for $z_{0ij}^{g1,g2}$ (the volume of domestic intermediates supplied by group $g1$ in sector $i$ to group $g2$ in sector $j$), we first assume that the share of intermediate inputs produced by $g1$ in sector $i$ equals the share of $g1$’s gross output in sector $i$. Then on the user side, we assume that $g2$’s share of intermediate input absorption in sector $j$ equals their share of intermediate inputs in total intermediate inputs demanded by the same sector. All this information is available in the firm census data. Based on these two assumptions, we split the original $z_{ij}^D$ according to the following formula:
\[ z_{ij}^{g_1,g_2} = \frac{x_i^{g_1} (x_j^{g_2} - v_i^{g_2})}{(x_j^{g_2} - v_j^{g_2})} z_{ij}^g, \quad (g_1,g_2 = SL, SS, FL, FS, PL, PS) \]  

(21)

3. To set the initial value for \( y_{0i}^g \) (total domestic demand for goods and services supplied by firm group \( g \) in sector \( i \)), we use the following formula:

\[ y_{0i}^g = x_i^g - \frac{x_i^g}{x_i} \sum_{j=1}^{N} z_{ij}^g - e_i^g \]  

(22)

Notice that the above procedures implicitly assume that the supply of intermediate products/inputs for domestic use from each firm type in a sector is proportional to their gross output in that sector.

6. Estimation Results

6.1 Contributions to China’s Economy

Based on the estimates of the model described in Sections 2 and 3, we portray the domestic segment of GVC in China. Table 1 shows the importance of different firm types in aggregate statistics. Besides VAX, all numbers are computed based on actual data from either 2007 and 2010 IO tables or the 2008 firm census. Columns (1)-(3) in Table 1 show that SOE account for 5%, 19%, and 9% of firms, value added, and employment of China in 2008, respectively. The relatively small shares of SOE are in part due to years of economic reform led by the Chinese government to privatize or let go SOE, especially the small ones in downstream sectors. SOE’s contributions to gross exports and value-added exports (VAX) in 2007 are 12% and 21%, respectively (columns (4)-(5)). The large difference between SOE’s contributions to value added and gross exports suggests that SOE have a higher share of indirect exports through other firms, compared to other firm ownership types. Notice that while SOE’s gross export share declined significantly from 12% in 2008 to 9% in 2010 (columns (6)-(7)), their share in VAX actually increased slightly from 21% to 22%. These opposite trends will be analyzed in greater detail below.

(Insert Table 1 here)

Table 1 also shows that SME are numerous and employ the majority of workers in China (column (1)). They account for 55% and 79% of China’s value added and employment in 2008, respectively.
In terms of gross exports, their contribution is much smaller – only 28% (column (4)). This low share of exports is consistent with the conventional view that most small firms do not export because of the potentially high fixed export costs.\textsuperscript{25} In terms of VAX, SME account for 42% (column (5)). The much larger contribution of SME to VAX implies that they have a higher share of indirect exports, through other types of firms. In terms of the aggregate gross exports and VAX, SOE and SME look similar, but both the share of gross and VAX by SME decreased from 2007 to 2010. We will reveal the key underlying differences in terms of their distributions across industries and the channels through which they achieve a high VAX ratio.

As expected, FIE are much more export-oriented. They are small in number, similar to SOE, but account for close to half of Chinese gross exports. Their share in total VAX is much smaller (only 27%), consistent with the literature that finds low domestic value added in Chinese exports, particularly in processing exports (Koopman, Wang, and Wei, 2012; Kee and Tang, 2015). To the extent that most of the processing firms are FIE, which include firms owned by investors from Hong Kong, Macau, and Taiwan (HKMT), the results are not surprising. Processing firms import a large fraction of intermediate inputs and are responsible for the final stage of production, by taking advantage of China’s low labor costs.

### 6.2 Value Added Exports (VAX)

Next, we use our extended IO tables to decompose VAX by firm type into direct and indirect VAX, based on both the forward- and backward-linkage approaches, as described in Section 2. We will first report results based on the forward-linkage approach, before reporting those based on the backward-linkage approach in Section 6.3.

(Insert Table 2 here)

Column (1) of Table 2 shows the estimated volume of VAX of different firm types. The VAX of SOE, FIE, LP and SME in 2007 are respectively 1446, 1841, 718, and 2942 billion RMB.\textsuperscript{26} The corresponding 95% confidence intervals, reported in parentheses, provide great confidence that our estimated VAX for different firm types are highly robust. Column (2) reports the standard error to mean ratio of each estimate. For 2007, the ratio ranges from only 7% for FIE to 32% for SOE, and are similar for 2010. The relatively smaller precision for SOE’s estimated VAX could be due to the larger

\textsuperscript{25} See Bernard et al. (2015) for a theoretical model and stylized facts based on US firm-level data.

\textsuperscript{26} 196, 250, 98, and 399 billion USD based on 2007 USD-RMB exchange rate.
variance in the underlying firm-level values across SOE within an industry, which will be verified below.

Column (3) reports the ratio of VAX to gross exports (the VAX ratio) for each ownership type. It is worth noting that both SOE and SME have the VAX ratio above 1. Specifically, the VAX ratios of SOE and SME are 1.17 and 1.02 in 2007, respectively. As a comparison, the VAX ratios of FIE and LP are 0.36 and 0.70. The finding of SOE’s VAX ratio being larger than unity confirms the results in Table 1 that SOE’s contribution to Chinese exports is much larger if measured in value added terms than in gross terms. Moreover, these findings contrast sharply with the evidence for developed countries, such as the United States, where large firms’ share in gross exports is usually higher than that in value-added exports (i.e., the VAX ratio is typically smaller than 1). In summary, the low VAX ratio of Chinese aggregate exports, as reported in the literature, hides substantial heterogeneity in VAX across firm ownership types and sizes.

Panel B of Table 2 shows the same set of estimates using the 2010 IO table. As reported, all but FIE experienced an increase in VAX. The increase was particularly sharp for SOE and SME. SOE’s VAX ratio increased by about 47% while that of SME increased by about 27% (column (4)). The significant increase in the VAX ratio of SOE lends some support to the anecdote that the state sector has advanced their prominence in the Chinese economy in recent years, especially after the global financial crisis in 2008 when the Chinese central government implemented macroeconomic policies to stimulate the economy. The higher-than-unity VAX ratios of both SOE and SME imply that many non-exporters from these two groups produce intermediate inputs and services that are embedded in Chinese exports.

Table 3 examines the potential reasons for the VAX patterns across firm types, by exploring how the VAX of each firm type was generated by selling to other firm types in the domestic input-output network. The estimated volume of indirect VAX through each firm type (column) is reported for a firm type’s indirect VAX (row). The corresponding share in the firm type's VAX is reported in square brackets. The 95% confidence interval of each estimate is reported in parentheses, with the corresponding standard error to mean ratio of each estimate reported in italics.

(Insert Table 3 here)

First, we find the following pecking order: SOE have the highest share of indirect exports in VAX, followed by LP and SME, with FIE having the lowest share. Specifically, in 2007, about 80% of exports from SOE are indirect (column (1)); the numbers increased slightly in 2010. In other words, 80% of SOE’s exports are values embedded in inputs used by firms that eventually export. The total
value of indirect VAX by SOE is about 1.15 trillion RMB, with 95% confidence ranging between 666 billion to 1.63 trillion RMB and the standard error to mean ratio of the estimate equal to 0.3.

For LP and SME, the indirect export shares are about 72% and 63%, respectively (column (1)). The indirect export share of SME increased significantly by 10 percentage points (from 63% to 73%) from 2007 to 2010, consistent with the hypothesis that small exporters could be financially constrained, especially after the global finance crisis, and are less likely to engage in direct exporting. Once again, FIE are very different from domestic firms and have a much lower share of indirect exports (about 46% in 2007, which decreased to 43% in 2010). Given the prevalence of FIE in processing trade and of intra-firm trade associated with vertical FDI, the low indirect export ratio is expected.

By splitting the IO table along the size and ownership type dimensions, we can also estimate the amount of indirect exports through different types of firms. As reported in Table 3, most of SOE’s indirect exports are through non-SOE. In particular, in 2007, FIE account for over 40% (35%/80%) of SOE’s indirect exports (column (3)), or 510 billion RMB. Based on 2000 random samples bootstrapped from firm census, we find that the standard error of this estimate is about 81 billion RMB, implying a 95% confidence interval ranging between 352 billion and 668 billion RMB. FIE’s contribution to SOE’s indirect exports further increased to over 55% (44%/80%) in 2010. On the other hand, SME account for 25% of SOE’s indirect exports in 2007, which declined to about 20% in 2010 (column (5)). Our extended IO table shows that SME’s contribution to SOE’s indirect exports is about 292 billion RMB in 2007. Based on 2000 random samples bootstrapped from firm census, the standard error of this estimate is about 53 billion RMB, implying a 95% confidence interval ranging between 189 billion and 394 billion RMB.

Both LP and SME also have high shares of indirect exports, but are lower than that of SOE. FIE also play a more significant role in helping LP to export indirectly, compared to SME. The role of SME in helping other firms export declined since 2007. For instance, when SME’s indirect export share increased from 2007 to 2010, the role of other SME in facilitating their own exports declined, with FIE taking up most of the increase. In summary, both SOE and LP have higher than average indirect export shares, with the former having a much higher VAX ratio. SME’s shares of exports, both direct and indirect, declined between 2007 and 2010, while SOE’s indirect exports increased, consistent with a rising VAX ratio as documented in Table 1.

Before explaining the different patterns of VAX across ownership types, let us discuss the large standard errors of some of the estimates, especially for SOE. Notice that the precision of the estimates for each firm type is naturally affected by the underlying standard errors of the corresponding firm-level values. For instance, if there are only a few SOE operating in an industry, bootstrapping the same number of observations with replacement may mean a high likelihood that the same firm is drawn
multiple times, yielding aggregates that could differ widely across bootstrapped samples for that firm type. In other words, the resulting standard error of an aggregate value across bootstrapped samples will be large.

Based on the values computed from different bootstrapped samples of firm-level data in the 2008 economic census, Table 4 reports the standard error to mean ratios of \( x^g_i \), \( e^g_i \), and \( v^g_i \) used in the linear constraints (eqs. (12)-(17)) of the optimization model. Notice that other values used to initialize the model are either computed indirectly using these three key values, including imported intermediate inputs by firm type \( g \) (\( z^{F,g}_{ij} \) in the objective function (11)), domestic inter-sector transactions between different firm types (\( z^{A1,g2}_{ij} \) in the objective function (11)), and domestic demand for firm type \( g \)’s output in sector \( i \) (\( y^g_i \) in the objective function (11)), as described in eq. (20)-(22). Their distributions are the major driver of the differences among the estimates from the 2000 extended IO tables estimated using bootstrapped samples. Other constants in those adding-up constraints are not differentiated across firm types and are obtained directly from the IO table at the sector level.\(^{27}\) They remain constant across bootstrapped samples.

(Insert Table 4 here)

As reported in Table 4, the standard error to mean ratios of major constants in the linear constraints of each ownership type in our optimization model are smaller than 0.05 in both years in our bootstrap exercise, with the exception of those for SOE’s value added, and LP’s exports and value added, which take the values of 0.162, 0.116, 0.117 in 2007, respectively. These findings are consistent with the high standard error to mean ratios of the estimated direct and indirect VAX for SOE and LP as reported in Table 3. The question is how to reconcile the small standard error ratios for SME’s initial values on the one hand, and larger standard error ratios for their estimated VAX as reported in Table 3? One possibility is that SME participate in GVC by selling primarily to SOE and LP in the domestic economy. The less precise estimates of the latter two groups’ VAX may lower the precision of SME’s estimated VAX.

Next we attempt to understand the reasons for the similarity in the VAX ratio between SOE and SME by examining the cross-industry pattern of indirect exports across different types of firms. In Table 5, we show that a substantial heterogeneity in indirect export shares (in total VAX) across 14

\(^{27}\) These include imports in sector \( i \) (\( m_i \) in eq. (17)), domestic intermediate inputs sold among industries (\( z^{D}_{ij} \) in eq. (14)), imported intermediate inputs sold among industries (\( z^{F}_{ij} \) in eq. (15)), and the demand for final goods in sector \( i \) that are either domestic or imported (\( y^D_i \) in eq. (16) and \( y^F_i \) in eq. (17)).
broad industries. “Upstream” industries, such as energy and mining; metal and non-metallic mineral extraction; electricity, gas and water supply; as well as financial sector all have very high indirect export shares (over 90%). One reason for their high indirect export shares is that the sectors with high indirect export share tend to be non-tradable, either by nature or regulated by the authorities (in the case of banking, only 5 major state-owned banks have been dominating different segments of the sector due to entry restriction to private firms). They export indirectly by providing essential intermediate inputs and services to downstream exporters. Thus, focusing only on gross exports in analyzing firms’ export participation can substantially underestimate their actual participation in GVC and thus the impact of trade liberalization on the economy.

(Insert Table 5 here)

In addition to the cross-industry variation, within a sector we also see a non-negligible variation in the indirect export share across firm types. For instance, in the “Light manufacturing” sector, the ratio of indirect to direct VA exports is 50% in 2007, one of the lowest, but the ratio for SOE is 75%. That ratio for SOE further increased to 92% in 2010. In wholesale and retail trade, while the indirect-to-direct export ratio is 24% (38%) for SME in 2007 (2010), it is 83% (61%) for FIE. These differences may reflect the predominance of small and medium sized private trade intermediaries, while FIE are less likely to be engaged in services (possibly due to policy restrictions) but more likely to be producing goods and sell them overseas either by themselves directly or through other domestic trade intermediaries. A casual observation shows that SOE tend to have a higher indirect export share in sectors that are associated with a lower average indirect export share, such as electronic equipment; while SME tend to have a higher indirect export share in industries that have a higher average indirect export share, such as energy and mining, and the financial sector.

To analyze these channels more systematically, we use the method proposed by Antras et al. (2012) to compute the upstreamness indices by firm type for each industry. Briefly speaking, the upstreamness index captures the average distance between an industry and final-good consumers. The appendix describes several important extensions we make to Antras et al. (2012), and how we use the extended IO table to compute such indices. Table A3 in the appendix reports the 240 upstreamness indices (40 industries x 4 firm groups), along with the industry’s upstreamness index computed based on the conventional IO table. The top 5 most “upstream” industries (out of 40) in China are “Extraction of Petroleum and Natural Gas”, “Mining of Ferrous Metal Ores”, “Mining and Washing of Coal”, “Production and supply of Electricity and heat”, “Processing of Petroleum, Coking and Nuclear Fuel”. The values of upstreamness for these industries range between 4 and 5, meaning that these industries
are on average 4-5 industries away before reaching final-good consumers. The bottom 5 “upstream” industries are “Real Estate”, “Health and Social service”, “Education”, “Construction industry”, “Public administration and social organization”.

Consistent with the high indirect export ratio, SOE tend to have the highest upstreamness index among all firms types within each industry, while SME tend to have the lowest upstreamness index, particularly for the least upstream industries. Fig. 4 plots the SOE’s, FIE’s, LP’s and SME’s upstreamness indices against the industry overall indices, which are estimated using the original IO table. The upstreamness indices for SOE (blue squares) are mostly above the 45-degree line, suggesting that SOE often command a more upstream position than other firm types in value chain, even in the same industry. SME, on the other hand, are often operating in the downstream of the value chain and therefore are much closer to final-good consumers.

6.3 VAX (Based on the Backward-Linkage Approach)

So far, we have been using the forward-linkage approach to estimate direct and indirect VAX by firm type, which involves summing up the entries of $\hat{A}_yB\hat{E}$ (in eq. (4)) horizontally along each row of the IO matrix. In this section, we show how to use the backward-linkage approach and answer the question: “For each dollar of Chinese exports (aggregate or by firm type), how much of it is ultimately coming from SOE, FIE, LP and SME?” While the forward-linkage approach focuses on the channels through which each firm type’s VAX (by sector or at the aggregate) is generated, the backward-linkage approach decomposes a country’s gross exports into its direct VA and indirect VA from different firm types. The decomposition can be done for each firm type as well. For example, SOE’s gross exports can be decomposed into its own direct VAX, but also domestic VA originating from all other upstream firm types, including other SOE, as well as other firm types’ VA embedded in inputs used to produce those exports.28 This decomposition exercise permits an analysis of the distribution of VAX across firm types embedded in each firm type’s downstream exports, complementing the forward-linkage approach that focuses on the “paths” of VAX.

Using this backward-linkage VAX measure, we provide another set of results to examine how the domestic VA in Chinese exports is distributed across firm types, and how the distribution changed between 2007 and 2010. As reported in column (1) of Table 6, of the 10 trillion RMB Chinese gross exports in 2007, 14% can be attributed to SOE, directly and indirectly; while the contribution by FIE,

28 Such a backward-linkage perspective aligns well with case studies of GVC of specific sectors and products, such as the iPod or iPhone examples frequently cited in the literature.
LP, and SME are 18%, 7% and 29%, respectively. The findings of high contributions by SOE and SME to China’s exports resonate well with the findings that both types of firms have high VAX, as reported in Table 2. Foreign VA in Chinese exports in 2007 is 32%.

We also decompose each firm type’s gross exports into contributions by different firm types’ indirect exports. For instance, as shown in column (2), we find that for each dollar of SOE’s gross exports, SOE themselves contribute about 39 cents (24 cents directly and 15 cents indirectly), followed by 18 cents from SME and 10 cents from FIE. Imports account for 26 cents, lower than their contribution to China’s aggregate gross exports. Notice that the numbers along the diagonal of Table 5 are always the highest compared to other numbers in the same column, suggesting that each firm type contributes the most VA to its own gross exports, compared to other firm types.

(Insert Table 6 here)

The lower panel of Table 6 reveals that while Chinese gross exports increased by only 9.7% from 2007 to 2010, the contribution by SOE in terms of VA increased by 14.8%. Specifically, for each dollar of Chinese gross exports, 16 cents came from SOE in 2010, compared to 14 cents in 2007. SOE are not the only group that experienced an increase in VA shares between the two years. All three other groups also experienced an increase, at the expense of foreign VA (imports). These results are consistent with Kee and Tang (2015), who show using firm-level data that the increase in China’s domestic content in exports in 2000s were mainly driven by exporters substituting domestic inputs for foreign inputs. However, it is the SOE that experienced the sharpest increase in VA contribution, followed by FIE that had its VA share increased by 9.2%. Another fact revealed in Table 6 is that SOE’s VA shares increased for exports by all firm types. This is not observed for other firm types. For instance, FIE’s VA shares increased only for FIE’s exports but not for other firm types.

The backward-linkage approach can be used to distribute sectoral VAX in exports into different sources of firm types. Such an exercise provides another perspective to portray the cross-sector pattern of contributions by firm type. As reported in Table 7, a few sectors have more than 30% VAX originating from SOE. In 2007, these sectors include “Mining and Washing of Coal” (SOE’s share in the sector’s VAX = 39.98%), “Extraction of Petroleum and Natural Gas” (49.56%), “Mining of Non-Ferrous Metal Ores” (32.50), “Processing of Petroleum, Coking and Nuclear Fuel” (44.16), “Smelting and Rolling of Metals” (36.67), “Production and Supply of Electricity and Heat” (52.05). These are obviously “upstream” sectors that provide essential inputs to downstream exporters.

(Insert Table 7 here)
While SOE appear to have a dominant position in some sectors, they are not the firm group that has the highest VA shares for most sectors. It is the SME that often contribute more than 30% of VAX in most sectors. In fact, SOE’s VA share exceeded 30% in only 13 sectors (out of 40) compared to 24 for SME. For example, SME’s shares of VAX in “Foods and Tobacco” and “Manufacture of Textile Products” are 60% and 52%, respectively. These findings suggest that SME have been playing an important role driving Chinese exports. This is consistent with the hypothesis that a lot of SME do not export directly, possibly because of high fixed export costs. Instead, they participate actively by supplying intermediate inputs and services to larger downstream exporters. In 2010, the number of sectors in which SOE’s share in VAX exceeded 30% actually dropped from 13 to 11. However, in sectors in which SOE had the highest VAX share in 2007, SOE’s VAX shares have increased substantially. For example, in the “Mining and Washing of Coal” sector, SOE’s VAX share was 40% in 2007, which increased to 56% in 2010.

Finally, let us emphasize that our estimated IO tables are flexible enough to be easily modified to quantify not only different firm types’ VAX, but also other economic outcomes due to their participation in GVC. For instance, we can examine how much profits or employment are generated for different firm types in a country due to participation in GVC. The way to quantify the profit patterns is to simply multiply the right hand side of eq. (2) by the coefficient matrix of direct profits or employment. Similar to our analysis on VAX, we can also attribute export-related profits and employment for each firm type due to both direct and indirect exports. These estimation results are available upon request.

7 Concluding Remarks

This paper proposes a method to extend a standard input-output (IO) table to incorporate firm heterogeneity when portraying the domestic segment of global value chains in a country. Using conventional IO tables, firm-level data for both manufacturing and service sectors, and quadratic optimization techniques, we estimate direct and indirect value added exports (VAX) for different types of firms in China, and decompose a firm type’s indirect VAX into different channels through which they are realized. Our approach is flexible enough to incorporate standard bootstrapping of firm-level samples, which are used to compute standard errors and confidence intervals for the estimates in the extended IO table, as well as the VAX estimates.

Implementing our method using Chinese data, we find that in China, both state-owned enterprises (SOE) and small and medium domestic private enterprises (SME) have much higher shares of indirect
exports and ratios of value-added exports (VAX) to gross exports, compared to foreign-invested and large domestic private firms. Using China’s IO tables for 2007 and 2010 respectively, we find evidence of increasing VAX ratios for all firm types, particularly for SOE. These findings suggest that while China is moving up the GVC, SOE appear to be still playing an important role in shaping China’s exports. These findings contrast with the conventional view that China’s export growth is largely driven by the foreign-dominated processing and labor-intensive exports.

Besides providing a general methodology for other researchers to study the domestic segment of GVC in other countries, our findings shed light on the pattern and consequence of privatization in China. In particular, years of privatization have led to the dominance of SOE, not only large firms, in the upstream sectors. We leave the exploration of the political economy factors behind such privatization outcomes for future research.
References


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Figure 1: Input-Output Table with Separate Transactions by Firm Ownership Type and Size

<table>
<thead>
<tr>
<th>Domestic Intermediate Inputs</th>
<th>Intermediate use</th>
<th>Domestic Final Use</th>
<th>Export</th>
<th>Total Gross Output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dim</td>
<td>Large SOE (SL)</td>
<td>Small SOE (SS)</td>
<td>Large FIE (FL)</td>
</tr>
<tr>
<td></td>
<td>1,2,..., N</td>
<td>1,2,..., N</td>
<td>1,2,..., N</td>
<td>1,2,..., N</td>
</tr>
<tr>
<td>Large SOE (SL)</td>
<td>1</td>
<td>Z_{SL,SL}</td>
<td>Z_{SL,SS}</td>
<td>Z_{SL,FL}</td>
</tr>
<tr>
<td>Small SOE (SS)</td>
<td>1</td>
<td>Z_{SS,SL}</td>
<td>Z_{SS,SS}</td>
<td>Z_{SS,FL}</td>
</tr>
<tr>
<td>Large FIE (FL)</td>
<td>1</td>
<td>Z_{FL,SL}</td>
<td>Z_{FL,SS}</td>
<td>Z_{FL,FS}</td>
</tr>
<tr>
<td>Small FIE (FS)</td>
<td>1</td>
<td>Z_{FS,SL}</td>
<td>Z_{FS,SS}</td>
<td>Z_{FS,FL}</td>
</tr>
<tr>
<td>Large Private (PL)</td>
<td>1</td>
<td>Z_{PL,SL}</td>
<td>Z_{PL,SS}</td>
<td>Z_{PL,FL}</td>
</tr>
<tr>
<td>Small Medium Private (PS)</td>
<td>1</td>
<td>Z_{PS,SL}</td>
<td>Z_{PS,SS}</td>
<td>Z_{PS,FL}</td>
</tr>
<tr>
<td>Imported Intermediate Inputs</td>
<td>Abroad(F)</td>
<td>1</td>
<td>Z_{F,SL}</td>
<td>Z_{F,SS}</td>
</tr>
<tr>
<td>Value-added</td>
<td>1</td>
<td>Y_{SL}</td>
<td>Y_{SS}</td>
<td>Y_{FL}</td>
</tr>
<tr>
<td>Total Gross Output</td>
<td>1</td>
<td>(X_{SL})^T</td>
<td>(X_{SS})^T</td>
<td>(X_{FL})^T</td>
</tr>
</tbody>
</table>
Appendix A

Extending the method by Antras et al. (2012) to measure industry upstreamness

To measure industry upstream based on our IO table with 6 sub-accounts, we need to modify the method proposed by Antras et al. (2012). First, we construct a 42x42 matrix for each firm type $g_1$ with the following elements

$$
\delta_{ij}^{g_1} = \frac{\sum_{g_2} a_{ij}^{g_1,g_2} X_{j}^{g_2} + E_{ij}^{g_1}}{X_{i}^{g_1}},
$$

where superscripts $g_1, g_2 \in \{SL, SS, FL, FS, PL, PS\}$ represent 6 firm types, $a_{ij}^{g_1,g_2}$ is the IO coefficient between a pair of firm-type-sector discussed in Section 2 in the text. $X_{j}^{g_1}$ and $X_{j}^{g_2}$ are gross output by group $g_1$ and $g_2$ in sector $j$, respectively. $E_{ij}^{g_1}$ represents exports from sector $i$ by firm type $g_1$ used in sector $j$ abroad.

When computing industry upstreamness, Antras et al. (2012) assume that the share of imports (and exports) of sector $i$ that is used by sector $j$ is the same as the share of domestic intermediate inputs of sector $i$ used by sector $j$. We improve upon their computation by relaxing both of these assumptions. First, in eq. (A1), we do not need to subtract imports from total intermediate inputs. It is because when we estimate our extended IO model, we already make the corresponding adjustment to deal with imported materials by having a separate $A^m$ matrix. In other words, our IO coefficients, $a_{ij}^{g_1,g_2}$, do not include imported intermediate inputs. Thus, we do not need to make the proportionality assumptions as Antras et al. (2012) to exclude imports from domestic intermediate inputs in our computation of upstreamness.

Second, when computing $E_{ij}^{g_1}$, we use data of exported intermediate inputs at the sector-pair level (ij) from China’s customs. To assign exported intermediate inputs to each firm type, we use the share of each supplier’s firm type in domestic inter-sector transaction volume (i.e., $\frac{\sum_{g_2} a_{ij}^{g_1,g_2}}{\sum_{g_1,g_2} a_{ij}^{g_1,g_2}}$) as the weight. For sectors that we do not have exported intermediate inputs from China’s Customs (most of them are service sectors), we follow Antras et al. (2012) and make the same proportionality assumption to obtain $E_{ij}^{g_1}$.

We also adjust for the change in inventory at the sector level carefully. First, we obtain inventory by firm type and sector. Then following the approach proposed by Antras et al., (2012), we subtract inventory from $X_{i}^{g_1}$ in eq. (A1). After obtaining a 42x42 block matrix of $\delta_{ij}^{g_1}$, we use eq. (4) in Antras et al. (2012) to compute upstreamness by sector and firm type.